# Descriptions of Image Intensity Structure via Scale and Symmetry

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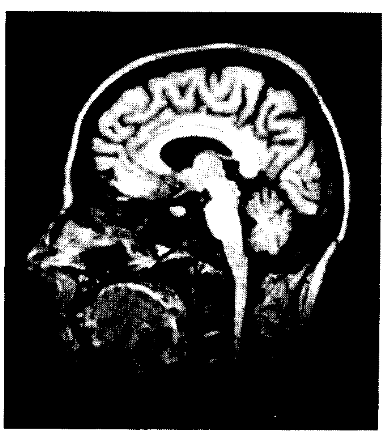
## ABSTRACT

As a means toward human definition of objects in 2D or 3D images, we describe an image analysis that provides a quasi-hierarchy of image regions in terms of which humans can quickly build object regions by an interactive approach. This hierarchy is generated by the ridge structure of the intensity surface corresponding to the image. These ridges in turn are defined via an intensity axis of symmetry (IAS), which forms a branching structure, in which the branches correspond to image regions and the parent/child relationships indicate ridge/subridge structure. The parent/child relationships are computed by following the IAS structure through changes in spatial scale, where scale change is achieved by a diffusion in which conduction may be related to edge strength.

## Introduction

A promising approach for the definition of objects in images is to have the computer analysis provide a framework in which a human who understands the semantics of the scene can quickly define the necessary objects using the image data. Such a strategy depends on automatically describing the image as a related collection of image regions that are coherent and visually sensible to the human user. More precisely, the image is described by a directed acyclic graph (DAG) of such image regions, with arcs specifying region containment. The examples given will be in terms of 2D images, but the approach applies also to 3D images.

Let our goal be to find image regions that match reasonably well the choices of humans not using the semantics of the image scene. We suggest that if the



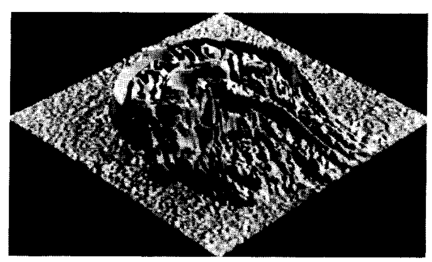


Figure 1. MRI brain image and corresponding intensity surface

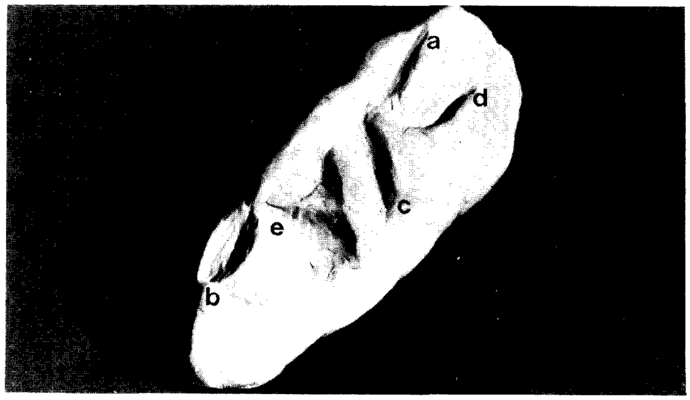



Figure 2. Ridge types: a) ridge fork, b) ridge fork and rejoining, c) monotonic ridge on flank of another ridge, d) nonmonotonic ridge on flank of another ridge, e) ridge connecting two other ridges.

image is viewed as an intensity surface, where height corresponds to image intensity (see figure 1), then shape properties of this surface will determine the region definitions. Humans seem to use ridges of intensities as organizing features for light objects on dark backgrounds, or courses for dark objects on light backgrounds. In this paper we will focus on regions lighter than their background, but all of the ideas will apply to the problem of finding regions darker than their background. The relation between these two sets of regions is also of interest if we are trying to imitate human visual behavior but is beyond the scope of this paper.

The objects that humans see are largely insensitive to rotations of the co-ordinate system, spatial scaling, and monotonic transformations of intensity. The analysis that we propose must have these characteristics.

The regions defined by a ridge included with its flanks do not correspond to the objects that we see in images, to the extent that a ridge flank goes down to a surrounding course, whereas objects stop at edges, which appear at the steepest places on flanksides. Our descriptions will therefore be made up of regions containing both an object and its nearby surround (not overlapping other objects). For the problem of region of interest selection for 3D volume rendering, which is our principal driving problem, the edges within the selected region come out in the display by virtue of volume rendering technique's property of relating opacity to intensity gradient [Levoy, 1988], so the ridge regions are entirely satisfactory to select an organ or other anatomic object for volume display. In those cases where determining the anatomic object edges is necessary, a simple means of deriving the edges from the boundaries of our ridge regions will be given in a later section of the paper.

Companion papers discuss algorithms for computing the regions and the DAG [Cullip, 1990] and tools for and the success of object definition in medical images with the use of this image description [Fredericksen, 1990].

## Ridges and Courses

Ridges are structures with a ridge top and two flanks falling to a course separating this ridge's flank from that of the next. The ridge top is defined by

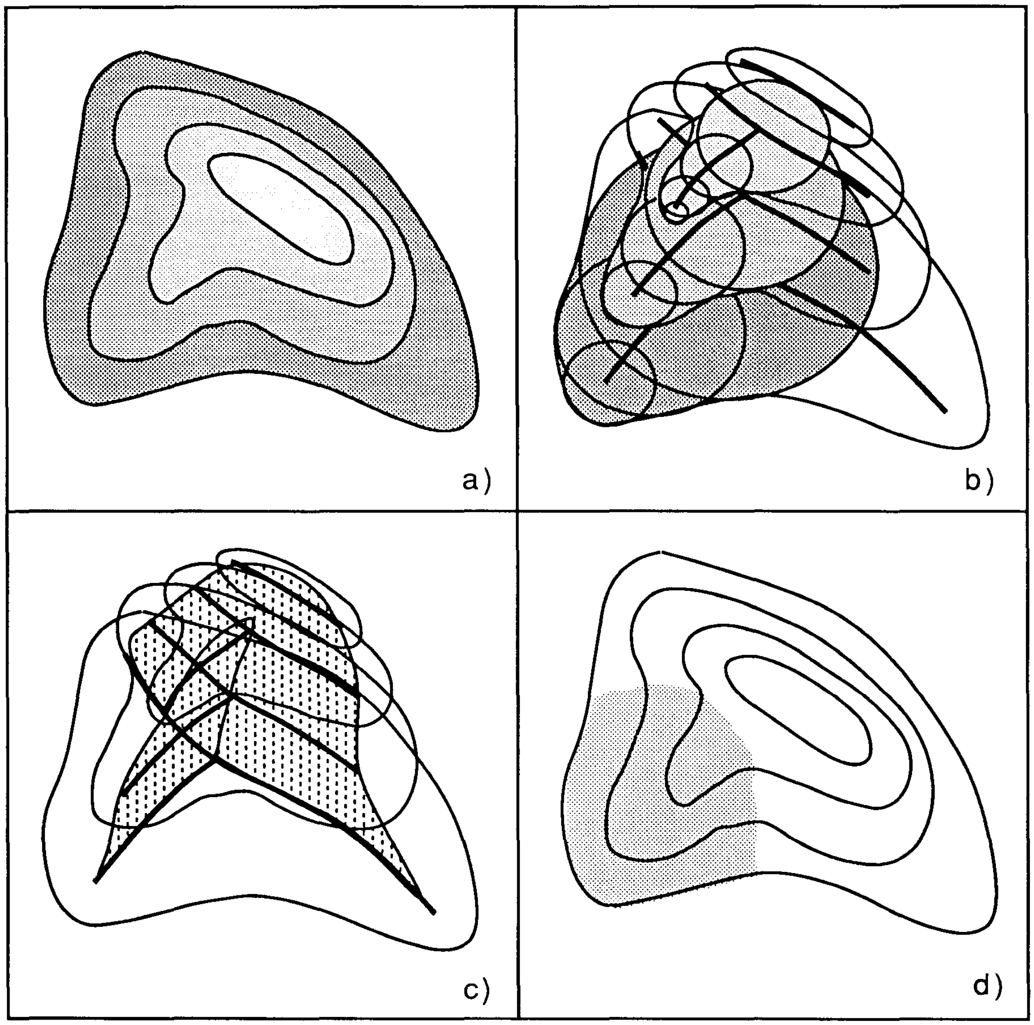



Figure 3. The IAS as a one parameter family of slice axes of symmetry. a) Level curves of a simple image; b) Level curves on intensity surface, their medial axes, and selected maximal disks; c) Level curves on intensity surface, and IAS; d) Image region associated with an IAS branch.

curvature properties of the surface there. As seen in figure 2, a) a ridge may fork into two ridges, and b) these two may rejoin, so that the pair surround an indentation with a course and even possibly a pit at its bottom. Furthermore, c,d) one ridge may appear on the flank of another. Finally, e) a ridge can begin on the flank of one ridge and end on the flank of another.

The result is that the intensity surface forms a directed acyclic graph (DAG) of ridges, with the parent relationship deriving from the child ridge being on the flank of or branching from the parent ridge. The fact that a ridge may fork and rejoin or connect two other ridge flanks mean that a single ridge can have more than one parent.

We suggest that a basic property of "ridgeness" is symmetry, so that for each point on one flank of a ridge there is another point on the other flank to which it has some sort of symmetry relation. Symmetry is an important visual property -- regions are two sided. We suggest that these two symmetries are related, that the ridge flanks are visually grouped to form the objects that we see, so that an analysis in terms of this symmetry will generate objects and hierarchical relationships that will provide a natural medium through which the human and computer can communicate about the image as objects are interactively defined.

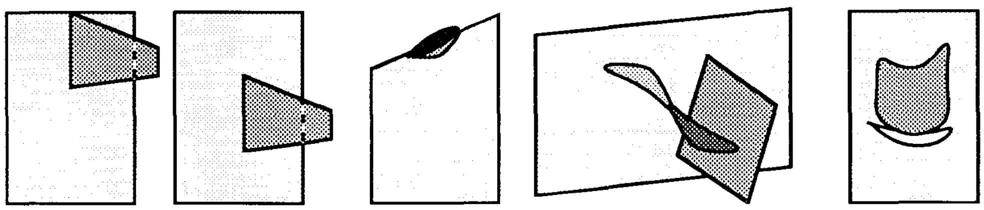
As a result we define an intensity axis of symmetry (IAS). This axis, made of a forest of branching or looping sheets, should fit under the ridges midway between the two symmetric flanks of the ridge. In order to avoid sensitivity to any monotonic transformation of intensity and also to avoid the incommensurability of intensity and space, the image (intensity surface) must be considered as a one-parameter family of slices in the intensity dimension, with the IAS made by stacking axes of symmetry defined for each slice. Thus, the IAS is the one-parameter family of medial axes of the intersections of the intensity surface with a series of slicing surfaces (see figure 3).

For now we have been slicing at isointensity levels, so that each axis of symmetry in the family is the medial axis of a level curve of intensity, even though this slicing focuses too greatly on intensity levels and too little on local image structure. The present IAS has the advantage that there is a 1-1 relation between each branch and a ridge top as defined by the locus of maxima of positive curvature of intensity level curves (these loci are called vertex curves) [Gauch, 1988, 1989]. However, it has the disadvantage that it is too directly tied to absolute intensity and ridge flanks go down only to the higher of the two courses bounding a ridge, so future consideration of slicing strategy is indicated.

As illustrated in figure 3, associated with each point on the IAS are (normally) two points in the image where the maximal disk centered at the IAS point is tangent to a level curve at its intensity. The basic sheets that are the leaves in the IAS quasi-tree (DAG) thus have associated with them a set of image points that are taken as the primitive regions of the image.

## Generating the Ridge/Subridge Relationships

Associated with a monotonic ridge on the flank of another ridge (its parent), then, is a branch of the IAS that grows from the IAS sheet lying under the parent ridge. If the child ridgetop branches from the parent ridgetop, the IAS child branch also begins at the top of the IAS sheet of the parent (see figure 4a); otherwise from the middle of the parent sheet (see figure 4b). A splitting and merging of the ridgetop results in a scoop in the IAS sheettop (see figure 4c), and a ridge connecting two other ridges corresponds to an IAS sheet connecting two other sheets (see figure 4d). The IAS under a nonmonotonic subridge forms a loop on the parent IAS sheet; at the intensity of the loop bottom the parent sheet tears (see figure 4e). More complicated forms arise if pits appear in inter-ridge valleys. In any case, we have the important relationship that although ridgetops may not form a connected structure, the corresponding IAS sheets do form a connected structure reflecting the subridge on ridge flank or ridge branching relationships. Because a given subridge may be on the flank of or branch from more than one other ridge, a given ridge may be the child of more than one parent. Therefore, the data structure describing the ridge connectivity is a DAG.



a) branch at sheet-top b) branch below sheet-top c) scoop in sheet-top d) connecting sheets e) loop & tear below sheet-top

Figure 4. IAS branchings

Each node in this DAG corresponds to a ridge, i.e. what we are taking to define sensible, coherent regions. The children of any such ridge are the subridges that describe sensible, coherent subregions of the parent ridge in question. In addition the parent ridge contains pixels that are on the parent's flank but not in any subridge. Unlike the pixels making up any subridge, this collection of pixels do not by themselves form a sensible, coherent region according to our definition. The result is that the DAG can be modified to have two kinds of nodes, those corresponding to ridges (sensible, coherent regions) and those collecting flank pixels not on a subridge. Each flank-only node has no children and a single parent, the ridge on whose flank it falls. Each ridge node has some number (possibly none) of (sub)ridge children, one flank-only child, and zero, one, or two parents (ridges of which it is a subridge). Remember that the user may eventually build semantically meaningful object regions out of combinations of ridge nodes that do not form a single ridge node. We have designed the image description with the idea that only combinations of ridge nodes will be necessary to build any semantically meaningful object regions. In particular, the flank-only regions will not make up such object regions, except as part of the ridge on the flank of which they fall.

Our problem is to decide which ridge is a subridge of which and to form the DAG which describes these relationships. Our strategy is to compute the region containment relations induced by the connectivity of the IAS branches and the annihilation of one branch into another as scale is increased (see figure 5). That is, one ridge is taken as a subregion of another ridge if the IAS branch corresponding to the former disappears into the IAS branch corresponding to the latter as the scale at which the image is considered is successively increased.

In the process of increasing spatial scale, it can be shown that a simple IAS branch (obtained by either ridge branching or by one ridge being on the flank of another ridge) will begin as a sheet but shrink from the top and bottom until it disappears smoothly into the sheet to which it is attached. This disappearance can be used to establish the child/parent relationship. Looping subsheets will normally disappear by narrowing down in their middle and breaking into two simple IAS sheets. An IAS sheet connecting two other sheets can detach first from one of its parents and then from the

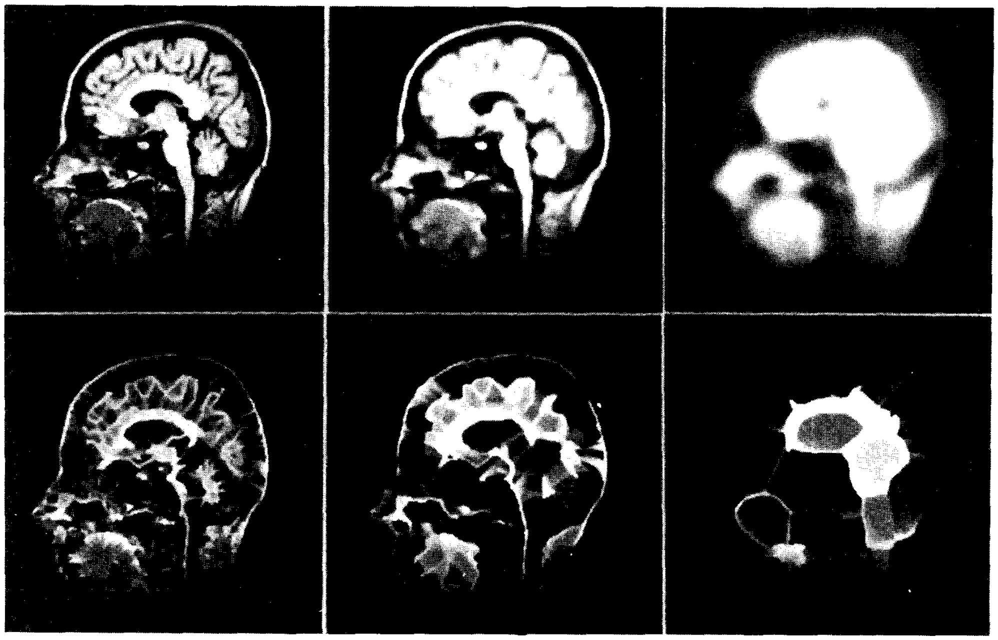


Figure 5. An MRI brain image at various scales and corresponding IAS's. Scale change was obtained by edge-affected anisotropic diffusion.

other, the intensity levels at which it is attached to one not being the same as those at which it is attached to the other.

## Scale Increase

Scale increase is performed by diffusion (Gaussian blurring), since this form of scale increase avoids most strongly the creation of structure [Yuille, 1983]. Simplicity suggests isotropic and stationary Gaussian blurring, but it seems preferable that the diffusion rate be matched to the scale of local objects. That is, diffusion across the interior of a large object should be faster than across a similar region made up of small objects or across the edge of the large object -- the scale change should be nonstationary. Moreover, diffusion along an edge should be faster than diffusion across it. To achieve this adaptively nonisotropic diffusion, we have begun using a program written by De Moliner [1989] at E.T.H., Zürich, in which a negative exponential function of intensity gradient magnitude (edge strength) is used as the local conductivity in a variable-conduction form of the diffusion equation [Perona, 1987].

## Edges

Regions chosen via the IAS do not normally consist only of the object of interest but also of a surrounding collar, since the ridges fall to the surrounding courses but the objects are normally thought to stop where the flank is steepest. As indicated earlier, these regions may sometimes be satisfactory. If regions bounded by the "actual" edge are needed, the (closed) boundary of the ridge regions can easily be used as the basis for an application of shrinking to the edge by an active contours approach [Kass, 1987] that pushes the contour up the ridge flank to the locus of steepest points that form the edge.

The IAS also provides the basis for defining an edge strength that reflects symmetry properties and for computing that edge as a continuous closed contour (in 2D) or surface (in 3D). Instead of the normal edge strength that is based on intensity steepness as the spatial arguments vary, we define the symmetry-dependent edge strength to be the steepness in intensity relative to the IAS. This works out as the average steepness in intensity across the two points ("involutes") sharing the same axis point, as the radius of the maximal disk touching the image points varies most strongly. That is, if $\nabla I_1$ and $\nabla I_2$ are the gradients at the disk touching points and $\nabla r$ is the gradient of the radius at both of these points, the symmetry-dependent edge strength is the component of $(\nabla I_1 - \nabla I_2)/2$ in the $\nabla r$ direction: $(\nabla I_1 - \nabla I_2) \cdot \nabla r/(2|\nabla r|)$. This is a sort of generalization to the IAS of Blum's object angle of the medial axis [Blum, 1978]. The two image points associated by a point on the IAS share the same steepness, so this strength can be computed simultaneously for the two flanks of a ridge. Moreover, the resulting edge is easily followed by following along the associated IAS sheet.

## Generalization to 3D

All of the above generalizes to 3D images, in which intensity is a function of three spatial dimensions. Ridges are still well defined (although there are three kinds of ridge/course rather than two), and the symmetry relationships generalize, with the tangent disks being replaced by tangent spheres. The branching structure complicates, but the diffusion remains quite the same as it extends to three dimensions. The details of this generalization are still to be worked out.

## Summary

An image's IAS followed through scale change by diffusion induces a DAG that subdivides the image into regions formed by ridges. These regions are designed to be sensible and coherent and thus form the basis for the definition of object regions of interest by an interactive user. Algorithms for computing the IAS, following it through diffusion, and producing the image-describing DAG of ridges, as well as their success in producing reasonable child/parent relationships, are described in the companion paper by Cullip et al. The design of a tool for using DAG descriptions for interactive object definition and the effectiveness of the DAGs resulting from the above procedures and the algorithms of Cullip are given in the companion paper by Fredericksen et al.

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