Salphasic Distribution of Clock Signals

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Abstract – **The design of a synchronous system having a global clock must account for the phase shifts experienced by the clock signal (clock skew) in its distribution network. As clock speeds and system diameters increase, this requirement becomes increasingly constraining on system designs. Two currently used approaches to this problem are to minimize skew by equalizing electrical path delays, and to re-cast system designs into an asynchronous (clockless) form. This paper describes a method that exploits fundamental wave propagation properties to minimize clock skews due to unequal path lengths for distribution system diameters typically up to several meters. The basic principles are developed for a loaded transmission line, then applied to an arbitrarily branching tree of such lines to implement a clock distribution network. Extension of this method to two- and three-dimensional distribution media is discussed.**

I. INTRODUCTION

Synchronous system design methodology, while well developed and widely used, is limited by signal propagation delays [1]. For system diameters that are small, signal interconnections can be considered equi-potential at any given instant. But the trend towards higher clock speeds and more system function are increasingly forcing designers to deal explicitly with time delays inherent in the propagation of signals on their interconnecting structures.

Some synchronous system organizations constrain data signal paths to be local, in an attempt to avoid propagation delay problems. The global clock signal, however, must be propagated across the entire system in a manner that preserves the correct ordering of events throughout the system. Considerable work has been done to address this problem, ranging from investigation of various clocking disciplines [2] to tuning the distribution network conductor lengths and amplifier delays [3], [4], [5] to minimize clock skew across the system.

A fundamentally different approach is to abandon the synchronous methodology altogether, in favor of self-timed and asynchronous delay-insensitive [6], [7], [8] disciplines. These techniques appear to promise scalability to any system size and speed, at the expense of additional hardware. For large future designs, these disciplines may become the mainstream methodology of choice. They are, however, substantially different from synchronous design methodology, and are neither widely understood nor practiced today.

This paper addresses the minimization of clock skew for moderately large synchronous systems, i.e., systems having clock distribution network diameters ranging up to several meters. This regime is seen as particularly applicable at the circuit board, backplane, and system levels rather than at the IC level. The results may be applied to clock distribution network design either alone, or in conjunction with other skew-minimizing techniques to further improve performance.

The method developed depends on properties of electromagnetic waves propagating on conducting bodies; in particular, on the general behavior of standing waves. As such, this method is potentially quite general, applying in principle to two- and three-dimensional conducting geometries as well as to wires. This paper presents an analysis of the useful special case of (one-dimensional) loaded transmission lines, and tree structured networks composed of such lines; and discusses the extension of the method to multi-dimensional conductor geometries.

Section II presents the theoretical groundwork for an ideal lossless transmission line and load. The mechanism whereby salphasic behavior (characterized by extended regions of constant phase with discontinuous phase jumps between regions) arises from standing waves is described, and the conditions are derived under which a finite loaded transmission line exhibits nearly salphasic behavior.

In section III, a canonical branch circuit is described which satisfies these conditions, and is used to show that an arbitrarily branching tree composed of such circuits also satisfies these conditions, thereby demonstrating the salphasic behavior of the entire tree.

Section IV describes the extension to multi dimensional distribution geometries such as “clock planes”.

Section V contains a summary and conclusions, and a brief discussion of future research needed to develop salphasic technology into a useful design methodology.

II. PHYSICAL PRINCIPLES

In an infinite lossless uniform linear transmission line, two waves V_f and V_r of equal frequencies propagating in the forward and reverse directions, respectively, are characterized as follows:

$$V_f = V_A sin(\omega t - \beta x), \tag{1}$$

$$V_r = V_B sin(\omega t + \beta x), \tag{2}$$

where V_A and V_B represent the amplitudes, ω represents the angular frequency, t represents time, x represents position along the transmission line, and β represents the phase constant or angular spatial frequency of the waves. Letting $V_B = V_A$ and adding the two waves at location x and time t on the transmission line provides an instantaneous voltage

$$V = V_A\big(sin(\omega t - \beta x) + sin(\omega t + \beta x)\big) = 2V_A sin(\omega t)cos(\beta x). \tag{3}$$

According to this relationship, the temporal phase ωt of the instantaneous voltage V is independent of location x. Thus, for any given value of x, only the amplitude of the sinusoidal wave is affected, while the phase remains constant.

This behavior, which we will call **salphasic**, provides that the phase of such a wave distribution is equal for all values of x within any region in which the sign of $cos(\beta x)$ remains constant. Salphasic behavior depends upon the equality of both the amplitudes and the frequencies of the forward V_f and reverse V_r traveling waves. These also happen to be the requisite conditions to produce a pure standing wave wherein the resulting voltage distribution on the transmission line varies sinusoidally in time but appears to remain stationary along the line. Therefore, any purely standing wave exhibits purely salphasic behavior.

Fig. 1 shows a finite lossless uniform linear transmission line having characteristic impedance Z_0, driven at location $x = 0$, and terminated by a load of impedance Z_l, at location $x = l$. If the transmission line and the load are both lossless, then the characteristic impedance Z_0 has no reactive component, i.e., $Z_0 = R_0 + j \cdot 0$, and the load impedance Z_l has no resistive component, i.e., $Z_l = 0 + jX_l$. Accordingly, the voltage reflection coefficient is [9]

$$\rho = \frac{Z_l - Z_0}{Z_l + Z_0} = -\frac{R_0 - jX_l}{R_0 + jX_l}, \tag{4}$$

which shows that $|\rho| = 1$. This satisfies the desired condition for a purely standing wave wherein the magnitude of the reflected wave V_B is given by

$$V_B = |\rho| \cdot V_A = V_A. \tag{5}$$

Therefore, a finite transmission line which is lossless and loaded by a pure reactance exhibits purely salphasic behavior.

On the other hand, if the finite transmission line and its load are lossy, in general $Z_0 = R_0 + jX_0$ and $Z_l = R_l + jX_l$. Along a lossy transmission line, the voltage varies according to the more general relationship [9]

$$V_x = V_A e^{-\gamma x} + V_B e^{\gamma x}, \tag{6}$$

where V_x is the voltage at any given location x, $\gamma = \alpha + j\beta$ is known as the propagation constant, and V_A and V_B are amplitudes of the forward and reverse waves, respectively. In this case, the following boundary conditions apply:
At the driven end $x = 0$,

$$V_0 = V_A + V_B \tag{7}$$

and at the loaded end $x = l$ [9],

$$\rho = \frac{V_B e^{\gamma x}}{V_A e^{-\gamma x}}. \tag{8}$$

Equations (6) through (8) can be solved for the load voltage, i.e., $V_l = V_x,\ x = l$:

$$V_l = V_0 \frac{\rho + 1}{\rho e^{-\gamma l} + e^{\gamma l}}. \tag{9}$$

If we let $\rho = e^{(\mu + j\nu)}$ as a notational convenience, it is clear that the low-loss load condition $|\rho| \approx 1$ is satisfied by an equivalent condition $\mu \approx 0$. Note also that the low-loss transmission line condition is $\alpha \approx 0$. In the limit as both losses become small, equation (9) becomes

$$\frac{V_l}{V_0} = \frac{\Big(cos(\nu - \beta l) + cos(\beta l)\Big) + j\Big(\alpha\big(sin(\nu - \beta l) - sin(\beta l)\big) + \mu\ sin(\beta l)\Big)}{cos(\nu - 2\beta l) + 1}. \qquad (10)$$

As α and μ approach zero, the imaginary component becomes negligible showing that in the limit V_l and V_0 are nearly salphasic. In particular with $|\rho| = 1$, as the real part α of the propagation constant γ becomes smaller, salphasic behavior holds for increasingly greater transmission lengths. Thus it follows that a lower loss line can maintain salphasic behavior over greater lengths.

To demonstrate salphasic behavior for a slightly lossy transmission line, a 12.7 Meter length of RG58/U type coaxial cable (Belden #9201) driven by a 100 MHz sine wave and terminated by a short circuit was simulated according to equation (9). Fig. 2 is a graph of the signal phase along the cable, computed from equation (9) as

$$\phi = tan^{-1}(Im\{V_l/V_0\}/Re\{V_l/V_0\}). \qquad (11)$$

Since the short circuit termination dissipates no energy, the reflected, or reverse-traveling wave has the same amplitude as the incident, or forward traveling wave at the termination. This closely satisfies the pure standing wave condition which results in strongly salphasic behavior near the termination. This is illustrated in Fig. 2 by the marked step-like shape of the phase plot near the termination (i.e., the lower right hand portion of the graph).

Due to the slight lossiness of the cable, however, the reverse traveling wave decreases in amplitude with increasing distance from the termination, while the forward traveling wave decreases in amplitude with increasing distance from the driving point. Thus the standing wave condition becomes progressively less well satisfied with increasing distance from the termination, resulting in progressively weaker salphasic behavior. This is illustrated in Fig. 2, where the step-like behavior becomes progressively softer with increasing distance from the termination.

For a very long cable, the phase-distance plot would approach a purely linear behavior far from the termination. On the other hand, for an ideal lossless cable, a perfectly sharp stair-step phase behavior would persist over the entire length of the cable because the traveling waves would remain of the same amplitude at all locations.

III. TREE TOPOLOGY AND ITS CANONICAL BRANCH CIRCUIT

Consider a lumped constant L-section having input series impedance Z_s, and output shunt admittance Y_p. Output voltage V_{out} is related to input voltage V_{in} by

$$V_{out} = V_{in}\frac{1}{Z_sY_p + 1}. \qquad (12)$$

Under the condition $Im(Z_sY_p) = 0$, the behavior of the L-section is salphasic, i.e., V_{out} and V_{in} are of equal or opposite phase. If $Z_s = R_s + jX_s$ and $Y_p = G_p + jB_p$ are nearly lossless, i.e., $R_s \ll X_s$ and $G_p \ll B_p$, then $Z_sY_p \approx -X_sB_p + jR_sB_p + jX_sG_p$. As $R_s \to 0$, the second term becomes negligible, and as $G_p \to 0$, the third term becomes negligible, leaving only the purely real first term. Thus, for nearly lossless Z_s and Y_p, a nearly salphasic relationship between V_{in} and V_{out} is maintained.

Fig. 3 shows a canonical branch circuit comprising a finite linear lossy transmission line of characteristic impedance Z_0, propagation constant γ, and length l, loaded by a circuit comprising a lumped series impedance Z_s, a lumped shunt admittance Y_0, and the equivalent shunt admittances $Y_1 \ldots Y_n$ presented by n similarly loaded canonical branch circuits connected to Z_s.

The equivalent admittances $Y_i,\ i > 0$ are determined using the following formula [9] for calculating the input admittance presented by a loaded transmission line expressed in terms of its characteristic impedance Z_0, its propagation constant γ, and the reflection coefficient ρ due to its load,

$$Y_{in} = \frac{1}{Z_0} \frac{e^{2\gamma l} - \rho}{e^{2\gamma l} + \rho}. \tag{13}$$

Hence, the aggregate output shunt admittance connected to Z_s may be represented in terms of the true lumped admittance Y_0 and the input admittances Y_{in} of each similarly loaded branch connected to Z_s. Thus, the load circuit is electrically equivalent to the L-section characterized by equation 12 if we let

$$Y_p = \sum_{i=0}^{n} Y_i. \tag{14}$$

Combining the finite lossy transmission line characterized by equation (9) and the load circuit characterized by equation (12) provides a voltage transfer function for the canonical branch circuit depicted in Fig. 3,

$$V_d = V_0 \frac{(\rho + 1)}{(Z_s Y_p + 1)(\rho e^{-\gamma l} + e^{\gamma l})}, \tag{15}$$

relating voltage V_0 driving this canonical branch circuit with voltage V_d driving the n canonical branch circuits connected thereto.

Under sufficiently lossless conditions, it was shown in equation (10) that V_0 and V_l are nearly salphasic, and in equation (12) that V_l and V_d are nearly salphasic; hence, V_0 and V_d are also nearly salphasic. Since this holds true for each canonical branch circuit, it holds for all voltages in an arbitrarily branching tree composed exclusively of such canonical branch circuits. Thus the voltages at all loads connected to the branching tree are salphasic with the driving voltage V_0 at the root of the tree and thus with each other.

A computer program based on equation (15) was used to simulate a model tree distribution network shown approximately to scale in Fig. 4. The model assumed an $18'' \times 18''$ standard multilayer glass-epoxy printed circuit board (PCB) with 2 $[oz/ft^2]$ copper cladding as the implementation medium for this network.

The branch circuit conductors are patterned on one layer of the PCB, separated from a ground plane by 11.8 mils of FR4 glass-epoxy dielectric. The simulated clock frequency was 40 MHz. The root branch and the vertical feeder branches were 20 mils wide, while the remaining branches were 10 mils wide. The loads, represented by a •, were each 10 pF. The numbers shown adjacent to each load represent the phase and magnitude of the voltage at the load relative to the voltage at the driving point, which was set to unit magnitude and zero phase.

These results show that all loads receives the voltage signal in approximately the same phase, even though their electrical distances from the driving point vary substantially. For example, the difference between the electrical distances from the driving point to the nearest and farthest loads is about 11″. This distance would correspond to over 20° clock skew in conventional systems. However, due to the near salphasic behavior of the signal, clock skew between any two of the loads is only 1.33° (93 pS).

Moreover, since eliminating signal reflections in a branching tree conductor geometry is infeasible, this topology is not useful for realizing conventional distribution networks. Therefore, conventional systems would require a separate conductor to each load, with all the conductors tuned to nearly the same electrical length, to achieve similar signal skew performance.

IV. GENERALIZED CONDUCTOR GEOMETRIES

The above development illustrates the fundamental mechanisms whereby standing waves exhibit salphasic behavior. This development was amenable to a simple mathematical treatment due to the one-dimensional mathematical nature of conventional electrical transmission lines, and the considerable body of existing applicable knowledge.

No simple, closed form mathematical methods are known for characterizing inhomogeneous wave solutions for the more general two- and three-dimensional cases of standing waves with arbitrary boundary condition geometries, behaving according to the wave equation

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = -4\pi f(\vec{x}, t). \tag{16}$$

Nevertheless, any linear system composed of a bounded, lossless transmission medium, with lossless linear loads, driven by a sinusoidal source exhibits pure standing wave behavior in three dimensions, and as special cases, in one and two dimensions. This can be seen through the following reasoning.

Since the system comprises purely linear components, no harmonics are produced from the sinusoidal signal. Therefore, the signal energy in the system is contained exclusively in sinusoidal waves at the signal frequency.

In the steady state, no net signal energy is exchanged between the signal source and the system, because the system is bounded and lossless. Thus, in the steady state, the wave propagating from the source into the medium carries an amount of energy which must be exactly balanced by the amount of energy carried by another wave propagating back into the source from the medium.

Under any given set of boundary conditions, the wave equation admits of only two such inhomogeneous solutions, which are identical in all respects except in their opposite directions of propagation. If the energies carried by these two waves are equal, their amplitudes must likewise be equal, thereby ensuring a pure standing wave. This remains true even after an indefinite number of reflections off the loads and boundaries of the system because the two waves are everywhere identical except for opposite directions of propagation.

Since the conditions for pure standing waves are the same as for pure salphasic behavior, the notion of salphasic behavior can be generalized to propagation across surfaces or through volumes, for signals behaving according to the wave equation, even though closed form solutions corresponding to the geometric boundary conditions may not be possible.

To demonstrate that salphasic behavior is indeed obtained in a lossless plane with lossless loads, two simulations were conducted using the CAzM$^{(TM)}$ program to perform finite element analysis on a model depicted in Fig. 5(a). This model represents a two dimensional conducting clock plane, adjacent and parallel to but separated from a conducting ground plane by a dielectric, as in a multilayer circuit board. A sinusoidal voltage is impressed at an arbitrary driving point, and the loads are connected at arbitrary locations to the clock plane. The simulated physical parameters for an ideal two-layer PCB and ideal capacitive loads were chosen as follows:

board dimensions	=	16×12	[inch]
dielectric thickness	=	1/16	[inch]
dielectric constant	=	4.5	[ϵ_0]
dissipation factor	=	0	[1]
surface resistivity	=	0	[$\Omega/\square$]
load capacitance (ea.)	=	$5^{(Fig.\,5(b))}$, $500^{(Fig.\,5(c))}$	[pF]
clock frequency	=	$50^{(Fig.\,5(b))}$, $100^{(Fig.\,5(c))}$	[MHz]
driving point coordinates	=	(8, 0)	[inch]
load point coordinates	=	(2, 4), (6, 10), (12, 6)	[inch]

One simulation was run with a clock frequency of 50 MHz and loads of 5 pF, the other with a clock frequency of 100 MHz and loads of 500 pF. The resulting simulated voltage distributions across the plane are plotted as isometric graphs over their respective zero-reference planes in Fig. 5(b) and Fig. 5(c), respectively. A contour is plotted in Fig. 5(c) to indicate the locations where the voltage is zero.

Corresponding to the voltage distribution shown in Fig. 5(b), the simulated phase is everywhere zero, i.e., identical with the phase of the driving source, to within the numerical accuracy of the CAzM program (better than $1 : 10^5$). The amplitude distribution is not significantly affected by the presence of the 5 pF loads located as shown in Fig. 5(a).

Corresponding to the voltage distribution shown in Fig. 5(c), two isophasic regions (regions within which the signal phase remains constant) are apparent, separated by the zero voltage contour. To within the numerical accuracy of the CAzM program, the simulated phase everywhere in the region containing the driving point is 0.00°, while the phase everywhere in the second region is 180.00°. The presence of even very large (500 pF) loads affects the amplitude at their locations only slightly, as illustrated by minor peaks in the graph at these locations.

V. SUMMARY, CONCLUSIONS AND FUTURE RESEARCH

The notion of salphasic behavior arising naturally from pure sinusoidal standing wave signals on transmission lines was described. A canonical branch circuit comprising a low loss transmission line with a low loss L-section load was analyzed for overall salphasic behavior, and used to recursively implement an arbitrarily branching signal distribution tree. The salphasic behavior of such a tree was shown to be well approximated by realistically low loss circuits.

This provides sufficient evidence that salphasic behavior could be exploited to control clock skews in high speed synchronous systems having diameters such that sufficiently lossless conditions are preserved. This depends largely on the lossyness of the transmission lines which make up the clock distribution network.

Using the salphasic approach, it is possible to build low skew clock distribution networks with a minimum of attention to adjustments and tuning, although further improvement could be achieved by doing so. Indeed, a model of an $18'' \times 18''$ PCB with thirty-six 10pF loads predicts a clock skew of less than 93pS between any two loads at a clock frequency of 40 MHz, with no tuning or adjustments whatsoever.

The arbitrarily branching tree clock distribution topology allows the use of geometries which are infeasible to implement in conventional technology, due to the requirement for impedance matching to control undesired signal reflections. This allows for far simpler clock distribution trees since stub length design violations need not be considered.

Clock planes similar to power and ground planes embedded in a multilayer circuit board appear feasible and very attractive as far as design simplicity is concerned.

Further research is needed to develop the salphasic notion into a useful design methodology. Thorough quantitative experimental verification of the theory has yet to be performed, both for commercial transmission lines and for PCB runs. Effects of crosstalk from signal lines to the clock network should be investigated, as should effects of non-linear loads. For widespread application, suitable standard cells for IC clock inputs need to be developed to implement amplitude-insensitive comparators for sinusoidal clock inputs. If larger system diameters are to be used, mechanisms for dealing with phase inversions between regions may be necessary, and design tools for tuning sub-trees to avoid the phase-inversion transition regions are needed.

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