Image Description via the Multiresolution Intensity Axis of Symmetry

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Abstract

A fundamental approach for providing an image description in terms of visually sensible image regions is described. It involves a) the representation of the image by a structure that captures essential image information and then b) the definition of a hierarchy of components of that structure by the order of annihilation of those components as the image is continuously simplified by lowering the scale. The information-capturing "essential structure" is chosen so that image regions are associated with each structure component during the image simplification. To guarantee image simplification, successive Gaussian blurring is chosen as the means of scale lowering. We argue that an essential structure that describes shape in both the spatial and intensity dimensions will produce an image description most likely to be useful for computer or human specification of image objects. In particular, we suggest that the intensity axis of symmetry (IAS) satisfies all desirable criteria for an essential structure. With such shape-based essential structures the approach of image description via annihilation under image simplification becomes a very attractive paradigm.

Introduction

Any process for the definition and labeling of objects appearing in images benefits from transforming the original image data into a description in terms of visually sensible regions. With such a description a source of intelligence, be it a human interacting with the display of the image or a computer program exhibiting artificial intelligence, has a good basis for fitting the image information to its model of the world in order to recognize an object.

In this paper we discuss the approach of producing such a useful image description by measuring an essential structure in the image and following it to annihilation as the resolution of the image is reduced. We give five properties which ensure that the image description is well behaved for image analysis. We show the IAS is an essential structure satisfying these properties, and we demonstrate a method for computing it.

Early Multiresolution Analysis

The most popular models of the human visual system (Robson [17], Koenderink [8,10], Wilson [19], Ginsburg [6]) recognize that it preprocesses the image by analyzing it simultaneously at multiple scales. In computer vision Crowley [4] realized early that analysis at multiple scales could provide an important means of image description on which model-based pattern recognition could be based, and not just efficient analysis, as suggested by many (e.g., Burt [3], Rosenfeld [18]). Crowley based his analysis on various Difference Of Gaussians approximations to the Laplacian of the image. He followed peaks and ridges (or their negative counterparts) in this Laplacian image through many scales while keeping the energy of the blurred Laplacian operator constant. Describing the image involved locating the scale at which each peak appeared most strongly.

More oriented to simultaneous description of image features at many spatial scales is a blurring approach that simplifies the image and defines objects in terms of the *disappearance* of their features with simplification. The idea is that image objects are defined first by regions of large scale, with detail of these objects defined by regions of smaller scale. Regions of large scale are those that are retained as the image is simplified by reducing resolution (blurring), while small-scale regions disappear under less blurring. The description must also include the relation between small- and large-scale regions.

Witkin [20], Yuille [21], and Koenderink [8,11] each suggested that Gaussian convolution was the best form of blurring, since it guaranteed image simplification with blurring, i.e., was the only form of blurring that did not allow the local creation of new values of any linear function of derivatives of the image as the blurring proceeded. Thus, for example, neither local image intensities (0th derivatives) nor Laplacian zeroes are created by this process. Lifshitz [13] has shown that the required Gaussian blurring need be neither isotropic nor stationary for the simplification guarantee to be met, and he has suggested that variation of the parameters of the blurring Gaussian across the image could be used to reflect *a priori* or tentative knowledge about the scene.

Essential Structures and Their Annihilation

Using the notion of following image features through simplification, Koenderink [8] suggested that the following of intensity extrema and of iso-intensity paths through Gaussian blurring could define sensible image regions: you followed each extremum to annihilation with a saddle point and defined the region as those locations whose iso-intensity paths ran into the path in scale space tracked by the extremum (the *extremal path*). Koenderink and we realized that this approach could be used to form an image description made from a hierarchy of these regions, where regions lower in the hierarchy were of smaller scale and blurred into their parent regions in the hierarchy. Lifshitz [14] implemented and experimented with this approach.

We suggest that a most important feature of this approach was that image regions were defined by the annihilation of their extrema under blurring, or to take a more constructive point of view, by the creation of these extrema as deblurring was successively applied to the fully blurred image. In this paper we develop a generalization of this idea of creating an image description that is hierarchical by scale by following what we call *essential structures* to annihilation.

The concept is that an essential structure should be an image descriptor that has the following five properties:

1. It induces a subdivision of the image into regions.
2. It captures essential region properties, including the way intensity varies across it and the spatial properties of the region, i.e., its shape, and therefore the regions it induces are semantically sensible.
3. The structure relating image components does not change until a component annihilates.
4. It induces a hierarchy of regions by defining for each component the containing component into which it annihilates.
5. It is applicable for images of any spatial dimension.

The major difficulty with focusing on the multiresolution behavior of extremal paths is that the regions defined by this method do not adequately reflect the shape of structures in the image. Thus, we need to devise multiresolution shape descriptors which are applicable to images and satisfy the five criteria above.

The Intensity Axis of Symmetry

To accomplish this task, we view the image as a terrain map, with intensity as height. Since the intensity dimension is incommensurate with the spatial dimensions, we must treat height specially and view the terrain map as being made up of a continuous pile of binary images, each corresponding to a level slice through the terrain, and having value 0 where there is air and value 1 where there is earth. The image I(x,y) has thus been characterized as a sequence of binary images, with height (intensity level L) parameterizing the selection of the slice.

When the shape of each of these binary images is captured by an appropriate shape description, the family of these descriptions with L as a parameter forms an image description which reflects both the spatial and intensity aspects of image shape. The description should decompose the binary image into cardinal regions. One of the most attractive approaches for such decomposition has focused on axes of symmetry (e.g., Blum [1], Brady [2], and Leyton [12]). Of the various alternatives the symmetric, or medial, axis (SA) stands out by being a connected tree that by division at branch points induces a decomposition into regions. The axis is the center of a figure and its associated radius function specifies the locations inside the figure. With the shape of each of the binary images characterizing an image captured by the SA (or any other axis of symmetry), the family across L forms an image description called the *intensity axis of symmetry (IAS);* or the 'earthen' IAS since the SA of figures with value 1 is used. Since each SA endpoint corresponds to a boundary point of maximum curvature magnitude (a *vertex*), each IAS sheet corresponds to a curve of level curve vertices (a *vertex curve*). These vertex curves are simply tracks in the original image, corresponding to ridges or courses in the "terrain map" and hence reflect the branching structure of the IAS. In the following we consider how the IAS / vertex curve image description meets our essential structure criteria.

Region Definition. Gauch [5] has shown that because the SA varies smoothly with image intensity, the IAS consists of a collection of branching sheets, each such branch characterizing shape in both space and intensity of a corresponding part of the image (see Figure 1). Associated with each sheet in the IAS there is a radius function, and a region image R defined by R(x,y) = the maximum intensity level for which the radius function of a sheet point at that level includes x,y. Thus, the IAS induces a subdivision of the image into regions that also carry information on intensity variation within the region.

Region Sensibleness. The fact that the IAS branches reflect ridges and courses in the image seems to allow curving objects to be followed and prevents objects from breaking into unrelated pieces. Like the regions defined by intensity extrema annihilation, the IAS structure captures the behavior of critical points in the image (they are peaks, pits or branch points in the IAS), but is more oriented toward to a whole object rather than these isolated points. Thus, the IAS captures essential image properties and defines sensible regions.

Region Hierarchy. Furthermore, branches in the IAS shrink to annihilation under Gaussian blurring of the image I. By detecting the order of annihilation of these structures it is possible to induce a hierarchy on IAS sheets and their associated image regions (see Figure 2). The IAS for an n-dimensional image is an n+1-dimensional forest of sheets, a prodigious object to follow through image blurring. However, vertex curves mark the top of IAS sheets (see Figure 3), so they can be followed through image blurring and when a vertex curve annihilates, the IAS sheet that it corresponds to must also annihilate. Therefore, it is possible to compute the IAS only for the original image, and for each vertex curve annihilation to follow the corresponding IAS sheet to its branch curve. This sheet defines a region image R and specifies R as a subregion of the region image corresponding to the limb sheet into which it connects. The hierarchy induced by this image simplification involves only a selection among branch sheets of the IAS which are already in the form of a tree (or a forest of trees). Furthermore, the regions they induce are directly described in terms of intensity and spatial shape by the properties of the symmetric axis transform.

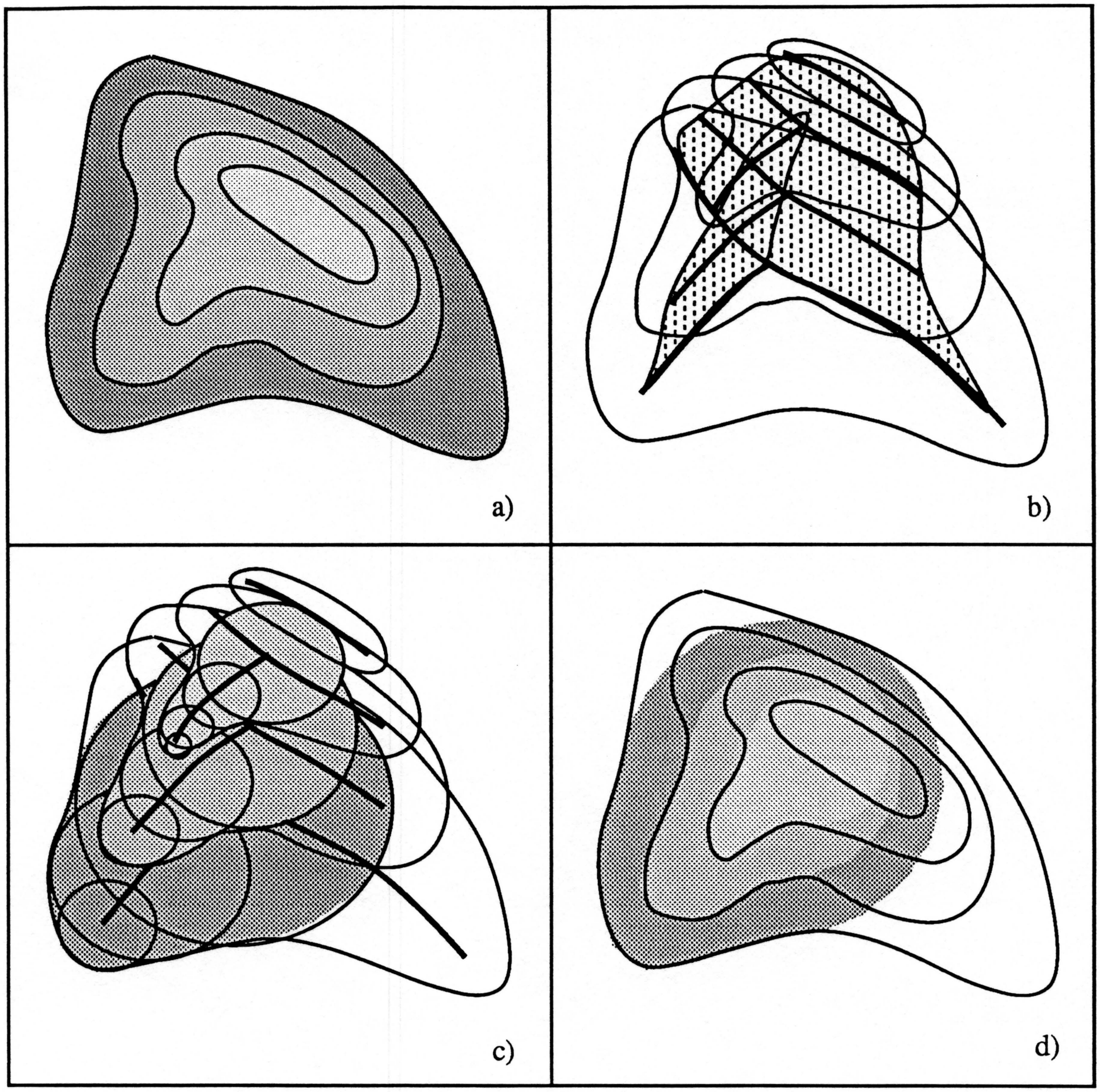

Figure 1: a) A simple grey-scale image represented by four level curves, b) its associated symmetric axis pile with the SA for each level shown in bold, c) the union of maximal circles centered on one branch of the IAS, d) the highest intensity at each point within this union yields an image associated with this IAS sheet.

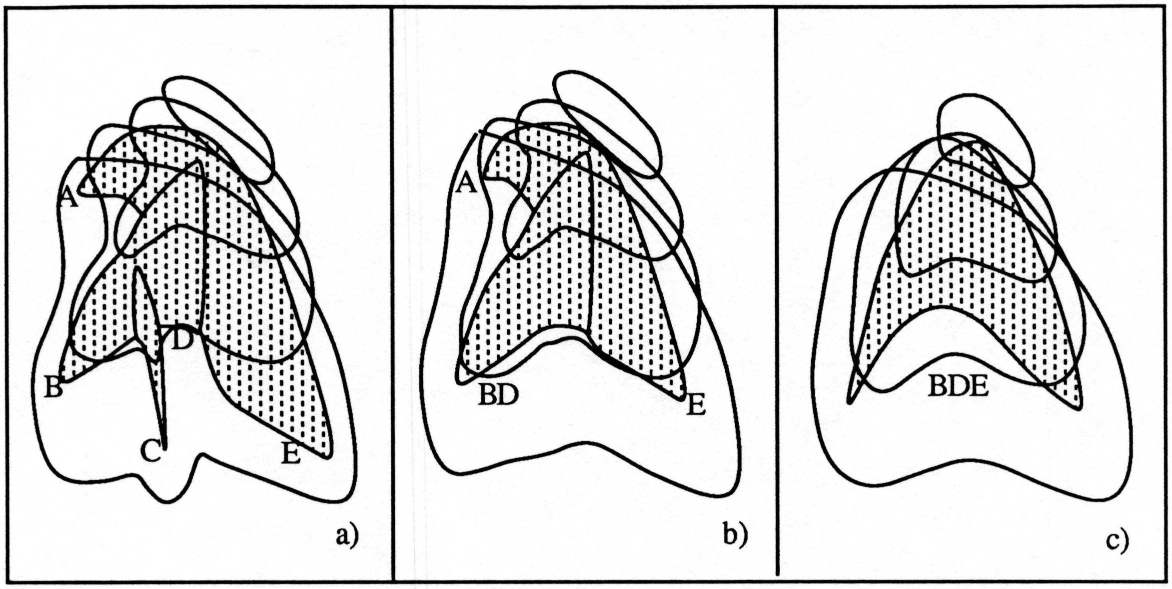

Figure 2: The effects of resolution reduction on the 'earthen' IAS of figure 2a are shown in 2b and 2c. The branch sheets appear under the ridges of the image represented by the level curves. When branch 'C' annihilates, we identify it as a subobject of the combined branch 'BD'. Similarly, branch 'A' is determined to be a sub-branch of 'BDE'.

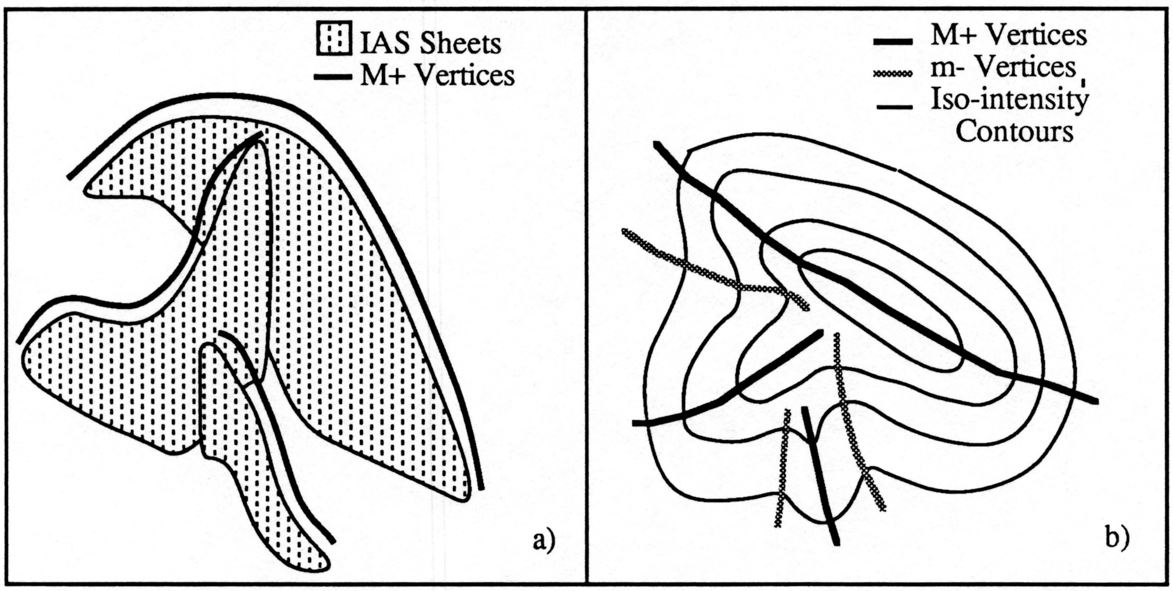

Figure 3: a) The relationship between the IAS for an image and the vertex curves corresponding to the end curves of the 'earthen' IAS. For clarity, only the vertex curves corresponding to positive curvature maxima (M+), are shown. b) Iso-intensity contours and vertex curves corresponding to positive curvature maxima (M+) and negative curvature minima (m-).

Consistent Simplification. Because vertex curves are used to study the multiresolution behavior of the IAS, we must consider the effect of Gaussian blurring on these curves. To locate vertex curves in an image, we calculate level curve curvature $K= v^t$ hessian(I) v, where I is the image and v is the unit vector in the direction of the level curve tangent, $(-\partial I/\partial y, \partial I/\partial x)$, and the first two derivatives of curvature K' and K" in the direction of the level curve tangent. Points where $K>0$, $K'=0$, and $K''<0$ are identified as vertex curves which correspond to the tops of IAS sheets. These curvature values are computed by the multiresolution n-jet approach of Koenderink [10]. This approach involves computing partial derivatives of the image $\partial^n I/\partial x^m \partial y^{n-m}$ for all n less than some limit, all $m \leq n$, and all degrees of blurring. We know that blurring does not cause new values of these derivatives to be created but what about K, K', and K"? These values can be shown already to range from $+\infty$ to $-\infty$. Thus, under image blurring no new values of level curve curvature are created. Unfortunately, the topology of the associated vertex curves can change. These changes occur when saddle-extremum pairs annihilate (or form) and also when locally concave or convex regions on the side of hills and valleys are destroyed. Fortunately, by following the smooth evolution of vertex curves through small increments of blurring, the simplification of IAS structure can be deduced.

Generalization to All Dimensions. This method seems extendable to higher dimensions, though details still need to be investigated. The notion of symmetry axes extends to higher dimensions by simply considering the locus of centers of maximal n-balls at each intensity level manifold. For example, if n=3 the axis characterizing each level surface L is the 3D SA described by Nackman [15]. The extension of vertex curves to higher dimensions is more complex because the number of types of curvature increases with the dimension of the manifold. For example, when n=3 surfaces have two types of curvature (Gaussian and mean). The extension of the IAS approach to higher dimensions (and temporal images) ought to be a subject of active research.

The IAS of the original image together with multiresolution vertex curves seem to satisfy all of the criteria specified for an essential structure and thus seems quite promising. Our next consideration is how to effectively compute these structures.

Calculating the Intensity Axis of Symmetry

While it is natural to describe the IAS in terms of the SA for a collection of binary images, there are several inherent problems with using this approach to calculate the IAS. In medical applications it is common for images to contain 12 or more bits of data per pixel. Thus, many slices would be necessary to represent image structures accurately, making the time to calculate the SAs for each slice considerable. More seriously, the boundaries of these binary images change topology as intensity varies. This makes it difficult to connect contours or axes from slice to slice. Finally, the simple thresholding techniques used to generate binary images often causes pixel artifacts in their corresponding SAs, and these complicate following the SAs from slice to slice. These problems together make it almost impossible to calculate the IAS on a slice by slice basis. Therefore, we must search for more global methods which process all intensity levels in the image simultaneously.

One way to accomplish this task is to extend the *active contour model* of Kass [7] to surfaces; thereby creating an *active surface model.* This will enable us to solve for the entire IAS structure simultaneously. When the original image is viewed as a surface in (x,y,I), it encapsulates the basic structure of the IAS. Thus, we use this surface as an initial approximation of the IAS. Then we calculate a function of (x,y,I) which reflects the symmetry of the image at each location and intensity level in the image. We use this symmetry function to "attract" the original image surface to the IAS surfaces. Finally, we determine the scale of each of the IAS sheets by following their corresponding vertex curves to annihilation under blurring.

Active Surface Model. The active contour model provides a technique for fitting closed curves to image features while maintaining internal constraints on the curve. These constraints are based on first and second derivative properties of the curve and ensure that the solution is smooth and well behaved, hence this technique is relatively insensitive to noisy image features. The key to this technique is the minimization of an energy functional. When the functional used by Kass for closed curves is generalized to surfaces, the equation becomes:

$$\text{Energy} = \iint [\, w_1 |f_u(u,v)|^2 + w_2 |f_v(u,v)|^2 + w_3 |f_{uu}(u,v)|^2 + w_4 |f_{uv}(u,v)|^2 + w_5 |f_{vv}(u,v)|^2 + g(u,v) \,]\, du\, dv$$

where $f(u,v) = (x(u,v),y(u,v),I(u,v))$ are the coordinates of the surface, and partial derivatives are denoted with subscripts. The five weights above control the effects of these partial derivatives and enable us to specify the surface behavior to be like a spline or like a flexible membrane. The attraction the surface by image features is given by $g(u,v)$.

While curves with several hundred points can be solved using the Euler equations described by Kass, the surfaces we are dealing with consist of tens of thousands of points. Thus, simultaneous solution via Euler equations becomes unreasonable and iterative relaxation techniques are more practical. In our implementation, each point on the active surface is examined on each iteration to see if any of the nearest neighbors in (x,y,I) has a lower contribution to the total energy than the current location. If so, that point on the active surface is moved to its new location. We say that the active surface has converged when the number of points moved in an iteration falls below a specified threshold.

Image Symmetry Function. The image-based constraint we use to direct the active surface towards the IAS is a function of (x,y,I) we call the *image symmetry function*. This function is defined to be $g(u,v,I)$ = distance from (u,v,I) to nearest point on the image surface at the same intensity. This follows the grassfire analogy used by Blum [1] to describe the symmetric axis. We compute the fire's quench points while ensuring smooth IAS structure by setting the initial active surface to be the same as the original image surface and using $-g(u,v,I)$ as the image-based component of the energy functional to be minimized on our active surface and iterating until convergence. Because the step size in our initial implementation is only one pixel, 50 iterations are often required before the active surface converges on the IAS. Variable step sizes like those used in simulated annealing might speed up this process while also ensuring that more global minimizations are found.

Using Vertex Curves. With a means of calculating the IAS for the original image, it or its corresponding vertex curves must be followed through decreasing resolution. Our initial attempts to follow vertex curves rely on the relationship between watershed boundaries and vertex curves. A watershed is defined to be the image region which drains to a common intensity minimum. Almost all major vertex curves in an image are also the tops of intensity ridges which act as watershed boundaries in the original image. The vertex curves which do not fall into this category are sub-branches of these ridges and can be processed separately. Therefore, it is possible to determine the scale of IAS sheets by following watershed boundaries through blurring.

We accomplish this by first identifying the intensity minima in an image and then following these extrema using a variation of Lifshitz's linking algorithm through a sequence of blurring steps until the minima annihilate with an intensity saddle point. Then we identify the parent watershed region to be the region which now captures the water from the annihilated minima. Thus, a hierarchy on watersheds is imposed. Finally, we identify the scale of each segment of watershed boundary (and hence vertex curve segments and their corresponding IAS sheets) by examining the scale of the two watersheds on either side of the boundary segment. This enables us to capture multiresolution behavior of the IAS without the excessive cost of recomputing the IAS for each level of blurring.

Results. The algorithms described above have been implemented in C under UNIX on both SUN and VAX machines and are part of a general purpose image processing environment at UNC-CH. For test images we have used digital subtraction angiograms of blood vessels because they contain long branching structures difficult to study using other methods. To illustrate the results of our calculations we display the "top view" of the IAS where the intensity at (x,y) corresponds to the brightest intensity of any IAS sheet which passes through the point (x,y). We have not yet developed tools which interactively use this image description to define image regions. Thus, we illustrate the multiresolution properties of the IAS by displaying the image, and its associated IAS, vertex curves and watershed boundaries at several levels of blurring (see Figure 4).

Discussion

We have described an effective method for capturing spatial and intensity aspects of image shape via the multiresolution IAS. However, the usefulness of this image description and some of its mathematical properties are still under investigation. The following problems must be addressed.

1. The dependence on intensity level curves seems unfortunate, and alternate means of slicing the image surface, perhaps reflecting image structure, need to be developed.

2. By definition, the 'earthen' IAS reflects the shape of white structures on black backgrounds. By considering the external SA of each intensity slice in our image (i.e., the 'air' IAS), we can derive an IAS which captures the shape of black structures on white backgrounds. How these two descriptions of the same image can be used simultaneously is an open question.

3. Hierarchies generated by the IAS share the inherent problems of the otherwise promising general category of methods based on annihilation of essential structures:

   a. Since these descriptions are defined only by image intensities, they cannot be expected always to reflect semantic information. Some means will be necessary either to edit the resulting descriptions to reflect such image understanding or to let such understanding affect the creation of the descriptions.

   b. An important weakness of the type of hierarchical description produced by following essential structures through blurring is its sensitivity to the order of essential structure annihilation. Similar images have descriptions that are made up of qualitatively different regions if their essential structure components annihilate in a different order. Others in our laboratory are working on this problem in an attempt to record sensitivity as part of the shape description.

   c. While the approaches we have sketched appear to apply to images of any number of spatial dimensions, it is not yet clear how to extend them to vector-valued images or to images of space and time.

Summary

We have shown that describing images hierarchically by following essential structures to annihilation is attractive if the essential structures satisfy a number of criteria. We have seen that the idea can be applied to a wide range of essential structures. However, the IAS / vertex curve essential structure seems particularly attractive in meeting all of the criteria. Other structures based on geometrical features of the intensity surface might also have these strengths.

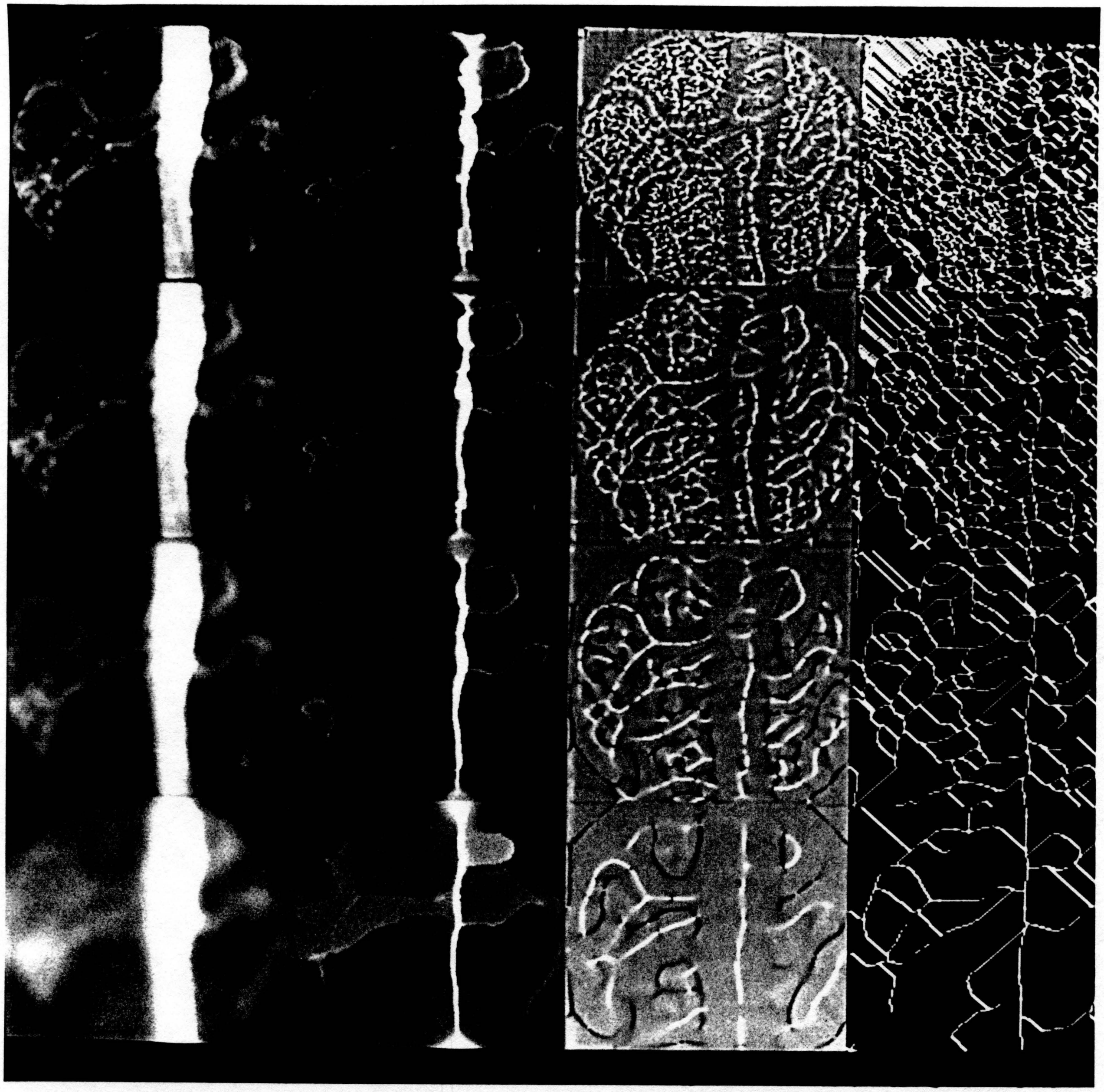

Figure 4: A sequence of blurred digital subtraction angiogram images (first column) with their corresponding IAS "top views" (second column) and level curve curvature (third column) and watershed boundary images (fourth column). In the curvature images magnitude of curvature is shown by the grey level; high positive curvature is shown in white, and low negative curvature in black. The vertex curves which correspond to the tops of IAS sheets are those with positive curvature maxima. Notice the consistent simplification of all four structures with blurring.

This paper has left many open directions for exploration, including how edges should be reflected, how cuts through terrain images should be made, how useful the vertex curve-based descriptions will be, and what other essential structures ought to be investigated. We are confident that such research will lead to the production of useful image descriptions.

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