Hierarchical Shape Description Via The Multiresolution Symmetric Axis Transform

July 1986

Technical Report 86-016

Stephen M. Pizer, William R. Oliver, Sandra H. Bloomberg

The University of North Carolina at Chapel Hill
Department of Computer Science
New West Hall 035 A
Chapel Hill, N.C. 27514

Submitted to IEEE Transaction on Pattern Analysis and Machine Intelligence

HIERARCHICAL SHAPE DESCRIPTION VIA THE MULTIRESOLUTION SYMMETRIC AXIS TRANSFORM

Stephen M. Pizer * +, William R. Oliver *†, Sandra H. Bloomberg *
Depts. of Computer Science *, Pathology †, and Radiology +
University of North Carolina, Chapel Hill, NC, USA

ABSTRACT

A method is proposed that produces a shape description in the form of a hierarchy by scale of simple symmetric axis sequences. An axis segment that is a child of another has smaller scale and is seen as a branch of its parent. The scale value and parent-child relationship are induced by following the symmetric axis under successive reduction of resolution. The result, in two or three dimensions, is a figure- rather than boundary-oriented shape description that has natural segments and is insensitive to noise in the object description. The general approach of hierarchy production by following a feature through successive resolution reduction will be presented, as will methods of resolution reduction and computer implementation. Also, the relation of this figure-based shape description to those based on boundary curvature will be briefly discussed.

1. HIERARCHICAL SHAPE DESCRIPTIONS

Shape descriptions are needed to allow models used in computer vision to reflect shape and to allow quantitative measurement of shape and shape change to be made in various applications such as medical imaging, biology, geology, and many others. The ideal properties of a shape description are that it naturally capture what we mean by shape and be automatically computable from the specification of an object by its boundary or its figure (the area or volume inside the boundary). Furthermore, the description approach should be applicable in both two and three dimensions.

Many methods of shape description have been previously proposed, among them many focusing on the description of boundary curvature [1-3], some based on a list of somewhat ad hoc features, some focusing on description of deformation from a primordial shape such as an oval [4-6], some focusing on description of the object figure, i.e., the area (or volume in 3D) of the object [7,8], and some focusing on both boundary and figure [9]. All except those based on the feature list have difficulty with the effects of noise in the figure or boundary definition and with separating detail from more essential shape characteristics, and some, e.g., the symmetric axis transform, have especial difficulties in this regard. Many produce descriptions of questionable naturalness.

To handle detail and noise naturally, one is led to a representation of the object as a hierarchy of segments at successively smaller levels of scale. For example, a human face might be described in terms of regions and subregions as in Figure 1. An advantage of this approach is that detail that may be noise is relegated to the lower parts of the hierarchy, and if it is to be seen as noise rather than important detail, it can be ignored without disturbing the description at higher levels of scale. Another advantage is that it allows top-down (large scale subobjects first) matching of models and descriptions of data objects. This approach of producing and using a hierarchical description has been taken with attractive results in regard to grey-scale image description by multiresolution methods which focus on intensity extrema [10,11].

The symmetric (or medial) axis transform (SAT) [7] has elegant properties in inducing segmentation of shapes into natural components, but its major flaw has been its sensitivity to noise in the boundary or figure specification. A related problem has been that no measure of the closeness of shapes fell out from the SAT, because there was no way to discern how to group parts of the axis into major components or how to measure the importance of a component. In this paper we show how the multiresolution and symmetric axis transform approaches can be married, producing a noise-insensitive hierarchical shape description with the natural segments that had been the original promise of the symmetric axis transform.

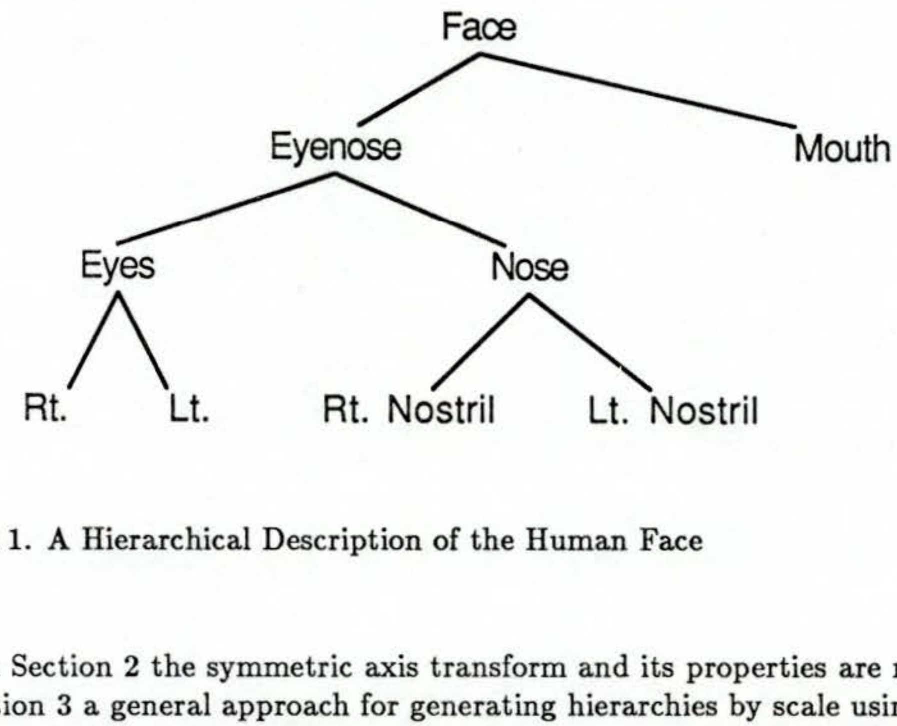


Figure 1. A Hierarchical Description of the Human Face

In Section 2 the symmetric axis transform and its properties are reviewed. Then in Section 3 a general approach for generating hierarchies by scale using multiple resolutions is presented, and it is applied to the symmetric axis to produce a hierarchical description of shape. Section 4 covers details of the multiresolution symmetric axis transform approach such as the means of reducing resolution and of following the axis as resolution is reduced. In the concluding Section 5 the relation between the proposed shape description and other methods is discussed, and extensions are suggested.

2. THE SYMMETRIC AXIS TRANSFORM

The symmetric or medial axis (SA) of a 2D object is intuitively the set of points within the object figure that are medial between the boundaries. More precisely, the SA is the locus of the centers of all maximal disks in the object, where a maximal disk is a disk entirely contained within the object figure but which is not contained by any other such disk. Figure 2 shows an example.

The SA forms a graph (a tree if the object has no holes). Segmenting the SA at branch points produces so-called simplified segments. Associated with each simplified segment is a part of the object figure made up of the union of the maximal disks of the points on the simplified segment. The object segmentation thus induced is frequently very natural. For example, in Figure 2 the bone shape is segmented into a rod and four knobs.

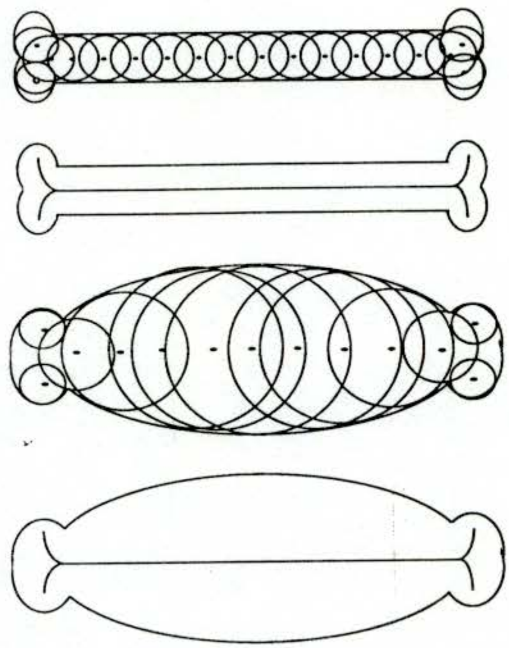


Figure 2. Maximal Disks in Simple Objects and the Corresponding Symmetric Axis

Associated with each point on a simplified segment is the radius of the maximal disk at that point. The SA together with this radius function of position on the axis is called the *symmetric axis transform (SAT)*. The radius function can be analyzed in terms of curvature properties [7] which characterize the behavior of the width of the object at that point, e.g., as flaring or cupping. One of the attractive properties of the SAT is that it separates these width properties from the curvature properties of the axis. These two sets of curvature properties can be used to further segment the SA and thus the object.

Nackman [8] has shown how the ideas of the SAT and the associated width and axis curvatures generalize to three dimensions. The "axis" becomes a locus of the centers of maximal balls, which in general is a branching surface. This surface can be subdivided into simplified segments at the branch curves, and again a natural subdivision is frequently produced.

The major weakness of the SAT is its sensitivity to properties of the detail of the object boundary. That is, changes in the figure or its boundary that are small in terms of distance can produce major changes in the SA (see Figure 3). A boundary feature that has a short arc length may result in a long symmetric axis branch, which moreover distorts the branch to which it is connected. The result is not only that branches that describe only detail are difficult to discern as such but also that major branches are split in such a way that a portion of axis that should naturally be viewed as a unit (a *limb* of the SA tree) is broken into unassociable portions. This weakness of the SAT has been so great as to destroy interest in it despite its otherwise elegant properties. Attempts to ameliorate it by pre-smoothing the boundary or by analysis, after SAT calculation, of properties such as axis arc length to boundary arc length ratios have foundered on the arbitrary thresholds that had to be imposed ("One man's noise is another man's detail."). However, we suggest that the imposition of a scale-based hierarchy on the symmetric axis segments solves these problems and thus allows one to take full advantage of the attractive properties of the SAT of inducing segmentation strongly related to our sense of shape and of separating width curvature from axis curvature.

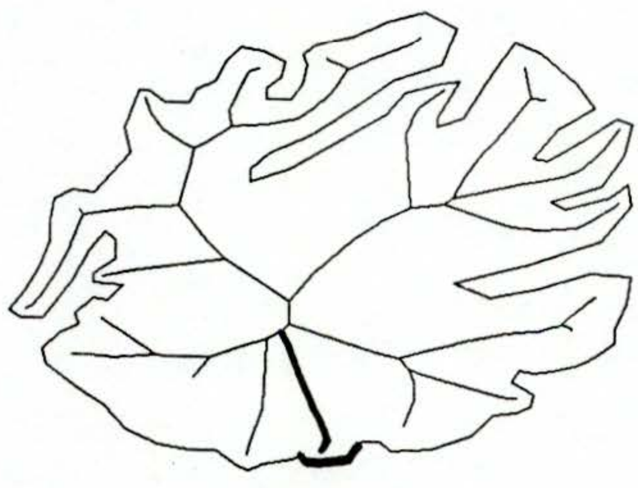


Figure 3. Sensitivity of the SA to Figure Noise. Note, for example, the large ratio of boundary to axis arc length in the region marked in bold.

3. THE SYMMETRIC AXIS BRANCH HIERARCHY

A useful paradigm for creating scale-based hierarchies for describing a complex distribution of components, such as an image or image object, is to find an important component feature that smoothly changes as the underlying distribution is blurred and that annihilates after an appropriate amount of blurring, then becoming part of another component. This approach has been fruitful with grey-level images, where, in a generalization of the pyramid approach, Koenderink [12] and Pizer [11] have suggested following intensity extrema under blurring until they annihilate. There, the amount of blurring necessary for a particular extremum to annihilate is taken as the scale of the extremum; in the process a region surrounding the extremum is associated with the extremum, producing its *extremal region*; in addition, when upon annihilation an extremum melts into another, the former is associated with the latter as its child in the hierarchy. The result is that the image is described by a tree of extremal regions, each labeled with a scale, where larger scale regions have tree descendants that are smaller scale regions contained by it.

We can apply this paradigm to the problem of describing object shape by focusing on the branches of the symmetric axis. We have found empirically that reasonable methods of smoothing of the object boundary cause the branches of the symmetric axis to change smoothly, such that at certain levels of smoothing a branch will disappear (see Figure 4). According to the general approach laid out above, we associate with the branch, as a measure of its scale, the amount of resolution reduction necessary to achieve annihilation, and we say that the annihilating branch is a subobject of the branch into which it disappears. This process is continued until only a branch-free SA remains.

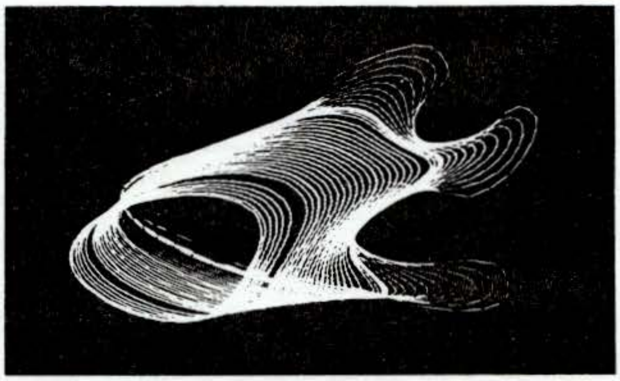
Every annihilating branch can be traced back to the part of the SA of the original, unblurred object from which it was smoothly generated. In the case of all but the branches at the frontier of the tree, these SA parts consist of a limb of the original SA, i.e., a sequence of simplified segments from which twigs have been removed at the one or more branch points where smaller scale branches that annihilated earlier were attached. The result is that the method has the property of defining naturally associated axis components from the associated segments in the sequence.

The complete multiresolution process defines a tree (hierarchy) of limbs and twigs (axis portions) in the original SA. The root is the portion of the axis to which the final branch-free SA traces back, and descendants of the axis portion at any node are the portions of axis which annihilated into that axis portion. The axis portion at each node is either a single SA simplified segment or a limb made up of a sequence (without branching) of simplified segments.

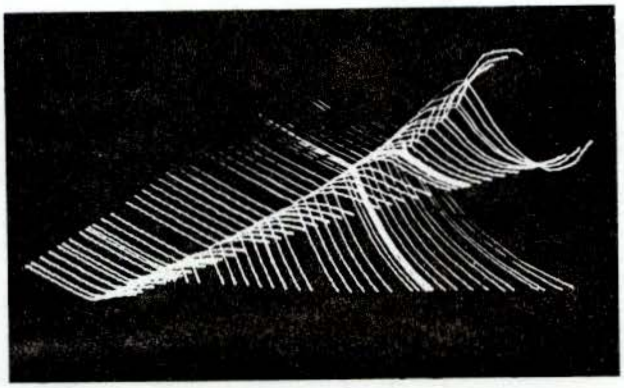
With every axis portion in the SA tree there is the original radius function on that axis portion. The union of the maximal disks centered at each point on the axis portion and with radii given by the respective radius function value is a subobject associated with the axis portion. The description tree can then be thought of as a tree of subobjects, of decreasing scale (but not necessarily area or volume) as you move down the tree. Each node in the tree (subobject) can be labeled by its scale together with properties describing the width (radius function) curvature and the axis curvature.

These ideas generalize straightforwardly to three dimensions. The axis components at nodes in the tree are simple surfaces, and the subobjects associated with a node are corresponding unions of maximal balls. Axis curvature and width curvature properties, as described by Nackman [8], as well as scale, label each node.

a.



b.



c.

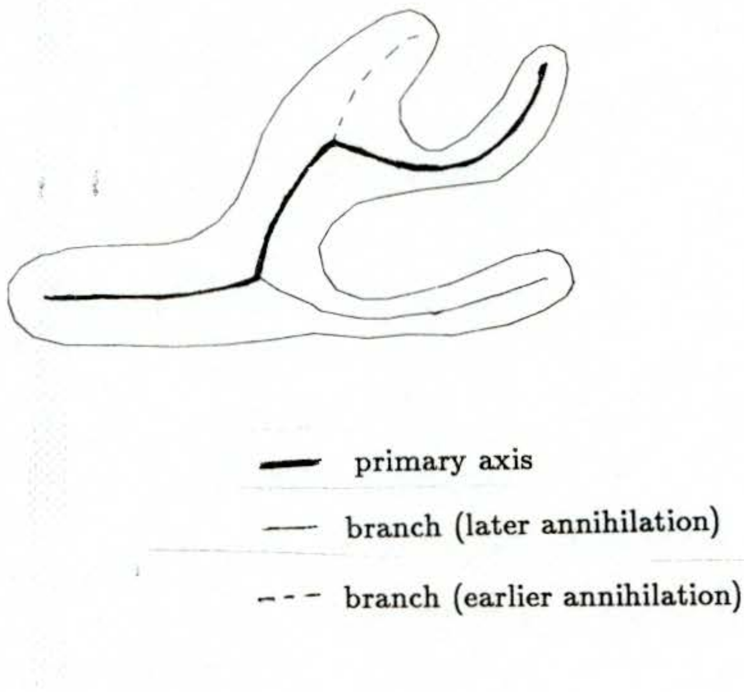


Figure 4. The SA Hierarchy based on B-spline Boundary Smoothing a) stack of boundaries, b) stack of SA's, c) induced description.

Examples given in Figure 5 suggest that this method produces natural descriptions. Furthermore, our experience is that objects that we see as similar, such as outlined human skulls viewed laterally, produce similar descriptions. Problems with this description arise from four facts. First, with some types of resolution reduction the topology of the figure and of the axis is not maintained: for example, a simplified segment of SA can split into two, or two can join into one. Second, shape features related to boundary concavities are not directly represented by this approach. In Section 4, where types of resolution reduction are discussed, we show how including other axes of symmetry in the representation seems to handle both of these problems.

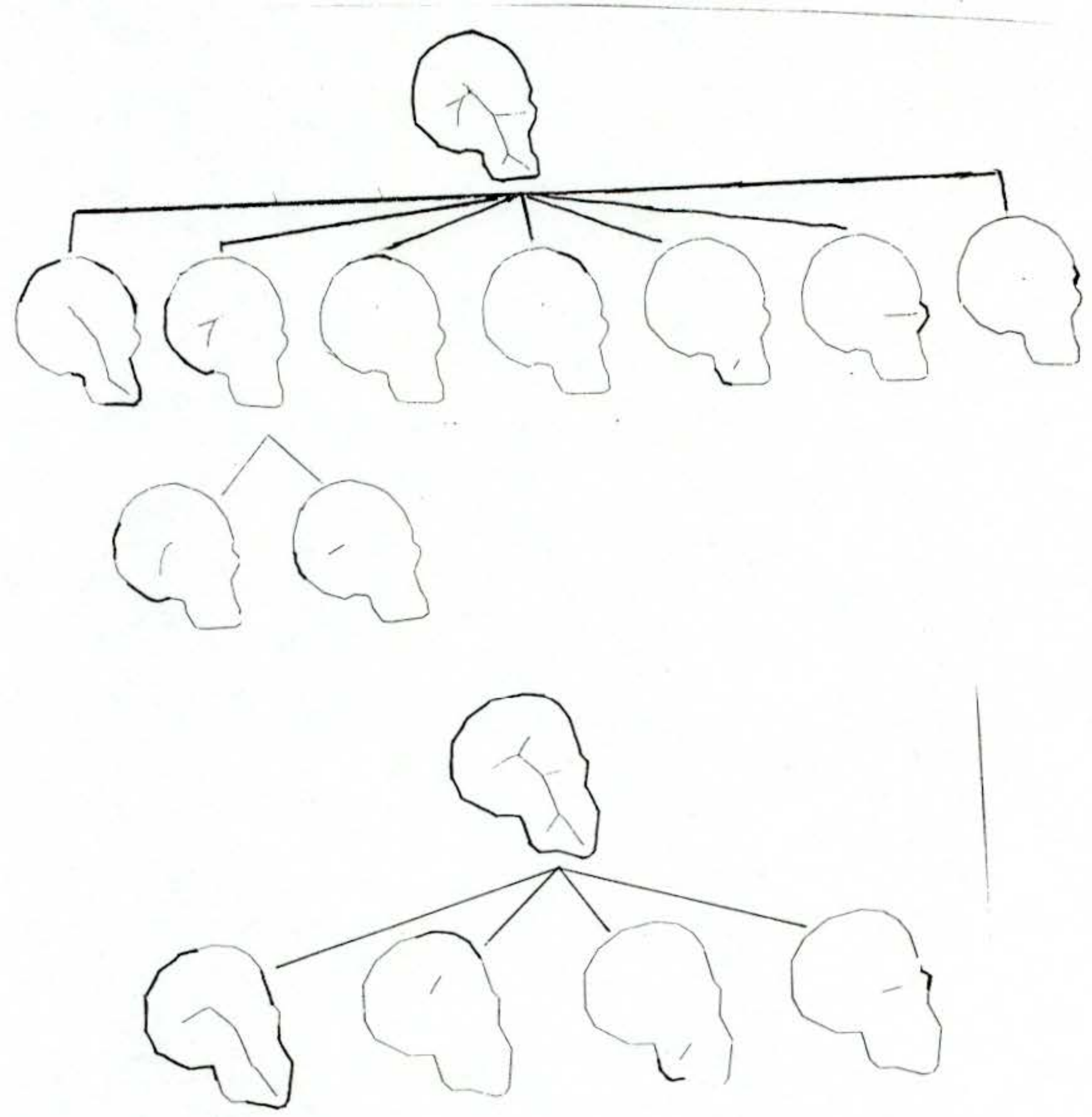
The third problem is that the sensitivity of the symmetric axis to small changes in the boundary can cause implementation difficulties in axis segment following. In particular, axis segments that do not exist at one resolution level can be artifactually created, or those that decrease smoothly in size for an underlying smooth boundary are artifactually removed at some resolution levels, only to reappear at later levels. Therefore, following the axis segments across steps is made difficult. This problem is discussed in Section 4.

Finally, there is one case in which similar objects have dissimilar descriptions. When the two branches emanating from a branch point are similar in scale in that when one annihilates the other is also almost gone, a small change in the scale (length or width) of one of the branches can change which of these two branches annihilates first and thus change which is considered part of the limb and which the attached twig. The tree changes that result from such small object changes are predictable and are discussed further in Section 5.

4. RESOLUTION REDUCTION AND SYMMETRIC AXIS FOLLOWING

What method should be used to continuously reduce the object resolution to produce the multiresolution stack of symmetric axes that induce the SAT hierarchy? The natural first thought is to focus on the boundary of the object to be described by applying some smoothing operator to its curvature. The result of such an approach, in which boundary points at one level of resolution were used as control points for a B-spline which forms the boundary at the next level, is shown in Figure 4. Koenderink [13] discusses why it is preferable to focus on first blurring the figure and then computing a consequently smoother boundary from the result, rather than directly to smooth the boundary. In essence the argument is that figure properties better capture the global relationships which we call shape than do boundary properties, which are too local. This very argument is the basis of the appeal of the symmetric axis method of shape description over methods based on describing boundary curvature.

Koenderink's suggestion [13] of the means for figure-oriented resolution reduction starts by treating the figure as a characteristic function, i.e., an image which is 0 outside the figure and 1 inside. He then would convolve the result with an appropriate Gaussian and compute a new figure boundary as a level curve in the result. Koenderink suggests that the level curve be taken at some fixed intensity, but this results in an arbitrary choice and causes the figure to shrink as it is blurred. Since it is commonly



a.

b.

Figure 5. Shape Descriptions Produced by the SA Hierarchy. The descriptions above are lateral views of two different human skulls. Each node of a description tree shows a component of the SA and shows in bold the part of the boundary corresponding to that component. The leftmost child of a node is its principal component axis. The other children of that node represent axis complexes branching from the principal component, arranged in order of decreasing scale.

understood that shape should be independent of size, we prefer to specify that the area (in 2D, volume in 3D) of the figure should remain constant with resolution reduction. Thus we choose the level curve which achieves this constant area (volume).

Resolution reduction based on figure blurring does indeed behave more intuitively correctly than direct boundary smoothing. Furthermore, it does have the additional advantage that it is in principle directly applicable to grey–scale object representations and not just characteristic function representations. However, there are two difficulties with this type of resolution reduction. First, topology is not maintained: connected components can split under blurring, disconnected components can join, indentations in the figure can turn into holes and vice-versa, and holes can disappear (see Figure 6). Second, with an implementation using a piecewise linear boundary the smoothness of the SA branch disappearance is more affected than with direct boundary smoothing. Let us discuss each of these difficulties in turn.

a.

b.

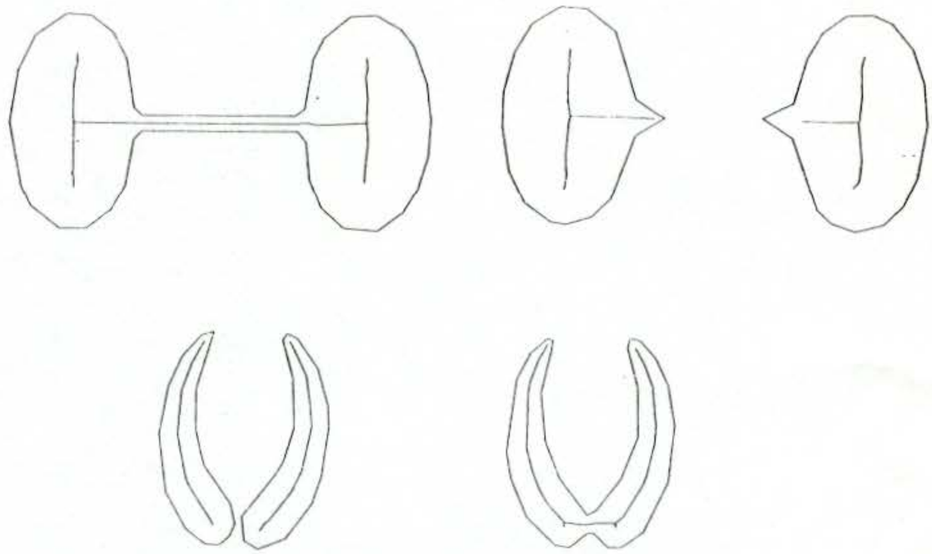


Figure 6. a) Splitting, and b) Joining of the SA under Resolution Reduction

From the point of view that the figure is essential and the boundary is derivative, the non-maintenance of topology is no problem at all. It is easy to argue that the object figures shown in Figure 6a are indeed close and that it is natural that a decrease in resolution should cause the isthmus between the two disks to be broken (eventually to be rejoined at yet lower resolution). Similarly, the object figures in Figure 6b should naturally combine as the resolution is lowered; other natural transitions are the closing of two nearby points of land around a bay to form a lake, the melting of a narrow strip between a lake and the sea to form a bay, and the drying up of a lake. We see it as unnatural to insist that topology be maintained under resolution reduction; instead, we must arrange our shape description not to be too sensitive to topology.

On the other hand, we are more disturbed by nonsmooth appearances or disappearances of large pieces of the SA. These appear to happen under some of the changes in topology listed above. For example, when two ellipses, each with a horizontal major axis and one just above the other, are blurred, a vertical segment of SA will appear nonsmoothly as the two ellipses join (see Figure 7a). We can avert many of these difficulties by including the external symmetric axis or the global SA as part of the SA.

The external SA of a figure is the SA of its complement. If we take the overall SA as the union of the internal and external SA (see Figure 7b), we find first that with resolution reduction as a piece of internal SA breaks, a corresponding pair of external axis pieces come together, and second that the overall SA now reflects concavities in the figure boundary directly. We therefore suggest that an improved shape description can be obtained by following this overall SA under resolution reduction. Note that as the resolution is reduced the object eventually becomes ovoidal and the external part of the SA becomes null.

The global SA [14] is formed by the locus of the centers of all disks tangent to two or more disconnected regions on the figure boundary. The ordinary (first order) SA and the external SA are subsets of the global SA. We have observed that when a new segment of ordinary SA appears nonsmoothly as two pieces of figure join or a hole is eliminated, it actually forms smoothly from a piece of global SA that is transformed into ordinary SA. On the other hand, when a hole fills in, the ordinary SA segment loop around the hole does disappear nonsmoothly. Further research is needed to catalogue these transitions and to determine the usefulness of computing the global SA.

The other disadvantage of resolution reduction based on figure blurring, as compared to direct boundary smoothing, is that following SA branches to annihilation is more difficult to implement using a piecewise linear boundary approximation. In our implementation the histogram of the image at each level of resolution is used to find the intensity such that the number of pixels with greater or equal intensity is equal to the original figure area (or volume). Points on the isointensity contour at that intensity are connected by linear segments, with the points selected by the recursive splitting method described in Ballard [15, Algorithm 8.1]. Our SAT algorithm essentially computes the Voronoi diagram of this collection of line segments and takes the SA to be the curves separating the regions in the Voronoi diagram, less those separating curve segments that touch the object boundary. The latter are removed because they are an artifact caused by using a piecewise linear approximation to a smooth object boundary. The result is an SA that is piecewise made up of linear and parabolic components.

The computed axis depends on the boundary approximation. In particular, small pieces of SA are either artifactually added or omitted depending on the approximation, and the added pieces are not necessarily small compared to SA pieces reflecting the "true" boundary at the given level of resolution, especially since the "true" branches get small as resolution is reduced. The artifactual addition or omission of pieces

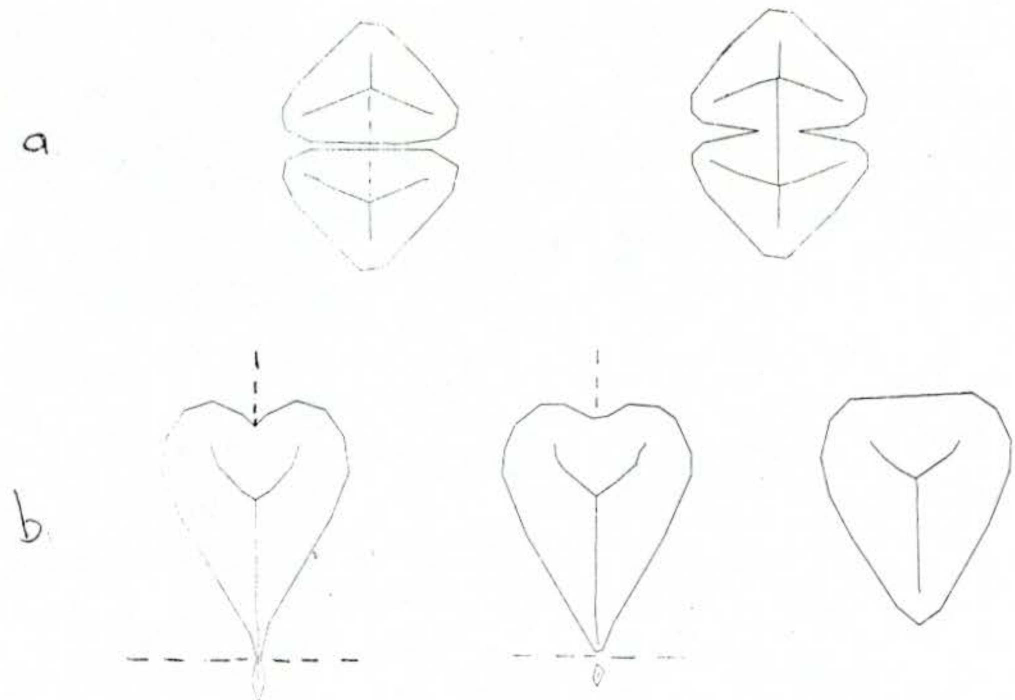


Figure 7. The Change under Resolution Reduction of a) The Ordinary (—) and One Other (- - -) Component of the Global Symmetric Axis of Two Nearby Joining Figures, and b) The Internal (—) and External (- - -) SA of Two Splitting Figures

that are unrelated across stages of resolution reduction complicates following the SA branches to annihilation.

This difficulty does not occur as greatly with direct boundary smoothing, as the boundary pieces at one step are related to those at the next. Nevertheless, the fundamental attractiveness of resolution reduction by figure blurring leads us to cope with its difficulty rather than resort to direct boundary smoothing. We believe that we need either to develop a method of figure-blurring-based resolution reduction in which the piecewise linear approximation at one step has related pieces to those in the previous step, or to replace the piecewise linear boundary approximation by a smooth piecewise approximation and develop a method of calculating the SAT for such approximations.

In our results to date, including those shown in Figure 5, the matching of SA segments from level to level has been done by hand, though the large majority of the matchings can be correctly made by a straightforward algorithm.

5. RELATIONS AND EXTENSIONS

Brady [9] has defined another form of an axis of symmetry that he calls *Smoothed Local Symmetries (SLS)*. He combines boundary curvature and figural properties in his definition. We find the boundary curvature aspects unattractive, as they involve an arbitrary degree of boundary smoothing, but the SLS, defined as the set of smooth loci of centers of chords that have angular symmetry with the boundaries they touch, is a set of axes that have some advantages (and some disadvantages) over the SA. We note here only that the multiresolution approach to inducing a hierarchy on the SA could equally well be applied to the SLS (in our modified definition).

The hierarchical SA or SLS descriptions have a relation to shape descriptions based on boundary curvature, such as the codons of Richards [3] or the boundary deformations of Leyton [6]. Codons describe simple convexities or concavities of the boundary. Leyton has shown [16] that these simple boundary segments have simple SA segments, i.e., correspond to the outermost branches of the overall SA. That work begins to relate the boundary and figure points of view, and more mathematics in this direction would be valuable.

Leyton's and Richards' methods, as well as others, describe the boundary as a sequence of curvature features or deformations to obtain them, but they frequently have difficulty determining an order of features or deformations to be applied. The multiresolution SAT method described above could be used to induce the order by scale for these methods, if the descriptions produced by latter are deemed superior to those produced by the multiresolution SAT.

Whether the multiresolution SAT is used directly to produce a shape description or as an order-inducing auxiliary to another method of producing a shape description, its weakness of having small changes in the image produce discontinuous change in the description must be dealt with. Recall that this behavior results from a close decision in deciding which of two branches or limbs emanating from a branch point forms the branch and which forms part of the branching limb. This behavior can happen at any branch point at which the two SA branches have almost equal scale. For example, in Figure 5a, the axis piece corresponding to the base of the skull is more prominent, while in 5b, the axis piece corresponding to the back of the skull is more prominent. It seems to be an application-dependent question whether trees differing in one or more of these close decisions should be considered instances of the same shape or of different shapes. In fact, it might be decided that the "close call" case is a single shape, independent of which way the call goes, while each of the cases, in either direction, of a "distinct difference" in scale between the respective branches might be called two yet different shapes. Of course, this decision would require an arbitrary threshold to be established between each of the shapes, and it also would require some means to be developed to define the scale of the non-annihilating branch that becomes part of the limb.

An alternative seems to be to let an annihilation cause both the annihilating branch and the "other" branch at the fork both to be declared as branches. The problem here is which of the (normally) three branches incident to a branch point to declare as the limb and which the two branches. Without a means to make such a declaration, the strength of the method in associating axis segments into a limb is lost. With such a means, one could detect when two branches are nearly equally strong and then produce alternative description trees corresponding to each of these branches annihilating first. Further research is needed on this question.

Finally, we expand on the attractive capability of the proposed method in describing shape of intensity-varying objects. The important implication of such a capability is that we can avoid the preliminary step of defining the pixels in the object before the shape is described, so the shape description can contribute to the object definition. Note that for every intensity level, we can compute the internal and external SA of the resulting level curves, and this pile of SA's can be considered a generalized SA of dimension n+1 if the original image is over n spatial dimensions. We are beginning work in the direction of producing a hierarchical description by following annihilations in the n+1–dimensional branches in this generalized SA as the image is successively convolved with a Gaussian.

In closing, we suggest that we have rescued the symmetric axis transform from being an interesting but useless approach to being a main challenger in the possibilities for shape description. However, more work is needed both to find mathematically how symmetric axes behave under figure- or boundary-based resolution reduction, and to make automatic the following of SA branches to annihilation, preferably by limiting the creation of artifactual branches and early omission of real branches.

In addition, we hope to have focused attention onto the multiresolution approach of following features to annihilation and thus inducing scale-based hierarchies. This appears to be a powerful method of deriving descriptions that are suitable for model-directed pattern recognition.

ACKNOWLEDGEMENTS

We are indebted to Fred L. Bookstein, Jan J. Koenderink, and Lee R. Nackman for useful discussions. We are grateful to Joan Savrock for manuscript preparation and to Bo Strain and Karen Curran for photography. The research reported in this paper was carried out under the partial support of NIH grant number R01-CA39060 and NATO Collaborative Research Grant number RG. 85/0121.

REFERENCES

[1] J. J. Koenderink, "The Internal Representation of Solid Shape Based on the Topological Properties of the Apparent Contour", *Image Understanding*, W. Richards, ed., MIT Press, 1985.

[2] C. T. Zahn and R. Z. Roskies, "Fourier Descriptors for Plane Closed Curves", *IEEE Trans. Comp. 21*(3): 269-281, 1972.

[3] W. Richards and D. D. Hoffman, "Codon Constraints on Closed 2D Shapes", *Computer Vision, Graphics, and Image Processing* 31: 265-281, 1985.

[4] F. Bookstein, B. Chernoff, R. Elder, J. Humphries, G. Smial and R. Strauss, *Morphometrics in Evolutionary Biology*, Special Publication 15, The Academy of Natural Sciences of Philadelphia, 1985.

[5] M. Leyton, "Perceptual Organization as Nested Control", *Biological Cybernetics* 51: 141-153, 1984.

[6] M. Leyton, "Smooth Processes on Shape", Tech. Rept., Department of Psychology and Social Relations, Harvard Univ., 1986b.

[7] H. Blum and R. N. Nagel, "Shape Description Using Weighted Symmetric Axis Features", *Pattern Recognition* 10: 167-180, Great Britain: Pergamon Press Ltd., 1978.

[8] L. R. Nackman and S. M. Pizer, "Three-Dimensional Shape Description Using the Symmetric Transform, I: Theory", *IEEE Trans. PAMI* 7(2): 187-202, March 1985.

[9] M. Brady and H. Asada, "Smoothed Local Symmetries and Their Implementation", Tech. Rept., Artificial Intelligence Laboratory, Massachusetts Institute of Technology, 1984.

[10] J. L. Crowley and A. C. Sanderson, "Multiple Resolution Representation and Probabilistic Matching of 2-D Gray-Scale Shape", Tech. Rept. No. CMU-RI-TR-85-2, Carnegie-Mellon Univ., 1984. Also see Crowley, J. L. and A. C. Parker, "A Representation for Shape Based on Peaks and Ridges in the Difference of Low-Pass Transform", *IEEE Trans. PAMI*, 1984.

[11] S. M. Pizer, J. J. Koenderink, L. M. Lifshitz, L. Helmink, and A. D. J. Kaasjager, *Inf. Proc. in Med. Imaging*, S. L. Bacharach, ed., Martinus Nijhoff, Dordrecht, The Netherlands, 1986, pp. 24-37.

[12] J. J. Koenderink, "The Structure of Images", *Biological Cybernetics* 50: 363-370, 1984.

[13] J. J. Koenderink and A. J. van Doorn, "Dynamic Shape", Tech. Rept., Department of Medical and Physiological Physics, Physics Laboratory, State University of Utrecht, The Netherlands, 1986.

[14] H. Blum, "3-D Symmetric Axis Co-ordinates: An Overview and Prospectus", draft of talk at Conf. on Representation of 3D Objects, U. of Penn., 1979.

[15] D. H. Ballard and C. M. Brown, *Computer Vision*, Prentice-Hall, Englewood Cliffs, N.J., 1982, p. 234.

[16] M. Leyton, "A Theorem Relating Symmetry Structure and Curvature Extrema", Tech. Rept., Department of Psychology and Social Relations, Harvard Univ., 1986a, submitted for publication.