

Intensity Mappings to Linearize Display Devices

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## Abstract

A method is developed for determining an intensity mapping to linearize the response of a viewed display device. When this mapping is applied to the intensity in a recorded image before the result is used to drive the display device, equal changes in recorded intensity become equally perceivable. Making this mapping part of the display device allows devices to be compared across images and allows the sensible application of intensity mappings meant to improve perception of image information for a particular image or imaging objective.

Over the past few years attention has begun to be given to problems of the display of images and not just to the means of detecting, recording, and processing them. This concern is based on the realization that the transformation from the recorded image, which is not directly viewable, to the displayed image, which is viewable, can limit significantly the information obtained by the observer. By image I mean a two-dimensional distribution of a single intensity variable. By the recorded image I mean the result of image detection, construction, or processing, for example the density distribution of silver on film, the electrical time signal which can drive a video display, the track of magnetic strength on videotape, or the array of numeric intensities in a computer memory. The intensity in the recorded image, which is called here the recorded intensity, serves as a driving intensity for a display device which transforms the driving intensity into a viewable intensity.

This paper is concerned with the performance of displays in transmitting contrast in recorded intensity, as opposed to their performance in transmitting spatial information as measured by properties such as resolution and geometric distortion. The images with which this paper is concerned are not ordinary pictures of scenes directly viewable by the eye but rather images such as infrared satellite pictures, radiographs, computed tomograms, sonograms, scintigrams, and thermograms, which are made by measuring the two-dimensional distribution or projection of some physical parameter. With these images the objective is to dis-

cern abnormal increases or decreases in intensity, but unlike ordinary pictures of scenes, these images need not be displayed so that contrasts appear realistic.

To optimize contrast transmission performance, we need to be able both to compare display devices across images and to arrange the display process so that intensity differences of special interest or significance are sensitively transmitted. The second objective requires us to design intensity mappings matched to an image or class of images, i.e. functions from recorded intensity to recorded intensity that distribute the overall sensitivity of the display on those recorded intensities in which it is important to discern changes. An example is the mapping called windowing (see figure 1), commonly used in computed tomography.

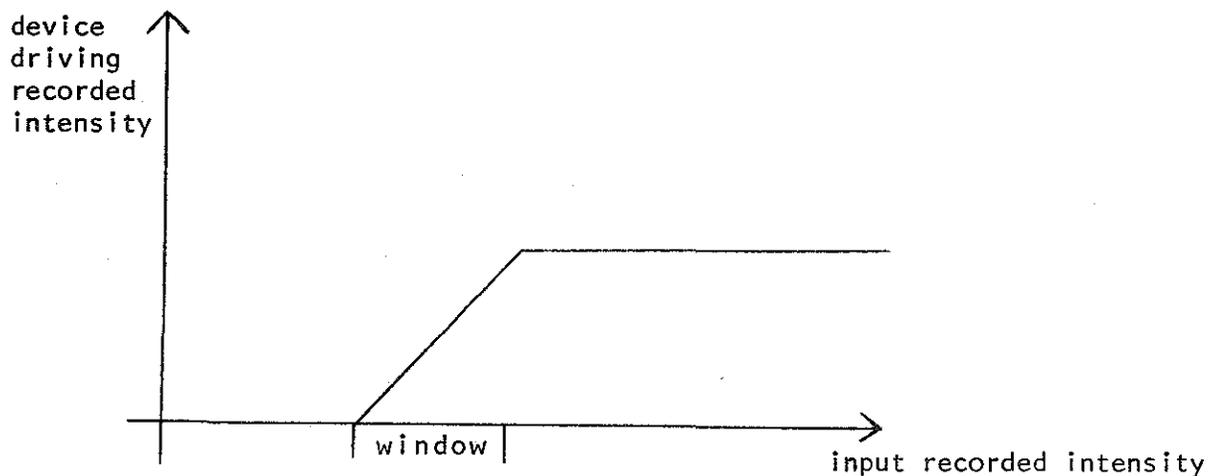


Figure 1: Windowing intensity mapping

Neither of the aforementioned needs can be satisfied if sensitivity as a function of driving intensity has different shapes from display device to display device. The reason for the ina-

bility to compare devices across images is that if for each of the devices there is a region of driving intensity where its sensitivity is superior to the other device's, each device will be more useful for some images, namely those with important intensity changes in the range where that device is more sensitive. The reason for the inability to design intensity mappings conveniently is that the device's variable sensitivity is in effect an unknown or at least a not easily usable mapping to be composed with the mapping to be designed.

Ideally a display device should be a linear transducer when viewed: equal changes in recorded intensity should be equally perceivable.<sup>1</sup> Then if nonuniform sensitivity is desired for a particular image, an appropriate mapping can be consciously designed and applied to the recorded image before it is input to the linear display device. Further, two such devices applied to any image could be trusted to have the same perceived effect, with the exception that the device with the greater overall sensitivity would present low contrast features (and image noise) better.

The objective of the research described here was to find a method for obtaining an intensity mapping for any device so that the modified device made of the intensity mapping followed by the

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<sup>1</sup> If some other standard relationship between recorded and perceived intensities is desired, such as having equal relative differences in recorded intensity be equally perceivable, one can easily produce this result by an extension of the technique described below.



the ambient illumination, the background intensity, the true positive rate defined to constitute detection, and the false positive rate setting the conservatism [Metz, 1978] at which the observer must work. Pizer and Chan [1979] have given these definitions and have given a relatively efficient methodology for determining the JND curve for any device and observer. They demonstrate that the variation in this curve across observers is fairly small, at least for grey-scale and heated-object-spectrum video display. For the purposes of the construction of intensity mappings to linearize the response of viewed displays, this variation can be ignored.

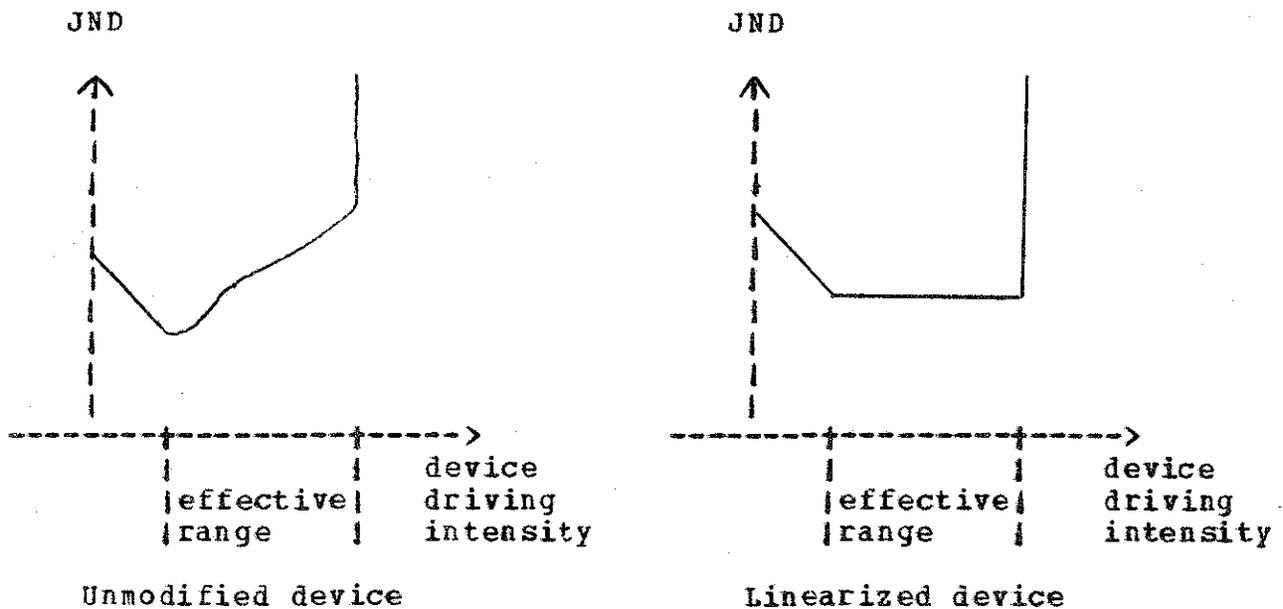


Figure 3: Just noticeable difference curves

Pizer and Chan showed that JND curves are always positive, with slope greater than or equal to -1. Furthermore, the slope is -1 at driving intensities below that at which changes can be perceived and is infinite at the intensity above which further changes are not perceivable (or perhaps not displayable). Let us call the range of driving intensity values for which the slope is strictly between -1 and infinity the effective range of the device. A linear device is one for which, in the effective range, equal changes in driving intensity are equally perceivable, i.e. one for which the JND curve is constant across the effective range (see figure 3).

From the JND curve one can determine a measure of the overall sensitivity of the display device. This measure is called the perceived dynamic range (PDR), of the device. Intuitively it measures the number of steps of 1 jnd between the bottom and top of the effective range,  $y_{min}$  and  $y_{max}$  respectively. That is, if the first step begins at driving intensity  $y_{min}$  and if, for all steps, when a step begins at intensity  $y$ , it ends (and the next one begins) at  $y + JND(y)$ , then the end of the PDRth step is  $y_{max}$ . More precisely, if we define a new type of step consisting of a group of  $N$  successive steps of successive values of  $(1/N) jnd(y)$ , the PDR measures the limit, as  $N \rightarrow \infty$ , of the number of these new steps within the effective range,  $[y_{min}, y_{max}]$ . Pizer and Chan showed that

$$PDR = \int_{y_{min}}^{y_{max}} \frac{JND'(y)}{JND(y) \ln(1+JND'(y))} dy \quad (1)$$

(for regions where  $JND(y)$  is constant, the integrand takes on its limit as  $JND'(y) \rightarrow 0$ , leading to  $PDR = (y_{\max} - y_{\min})/JND(y)$  if  $JND(y)$  is constant over  $[y_{\min}, y_{\max}]$ ).

In this paper it is assumed that the measured JND curve for the device is given and can be well approximated by a piecewise linear function:  $JND(y) = t_i + s_i y$  if  $y \in [y_{i-1}, y_i]$  (2) for  $i = 1, 2, \dots, n$ , where  $y_0$  is the left endpoint of the effective range and  $y_n$  is the right endpoint of the effective range (see figure 4). For the piecewise linear JND curve given in equation 2, substitution in equation 1 gives  $PDR = \sum_{i=1}^n PDR_i$ , (3) where  $PDR_i$ , the perceived dynamic range of the  $i$ th linear piece, is given by

$$PDR_i = \begin{cases} \ln \left( \frac{t_i + s_i y_i}{t_i + s_i y_{i-1}} \right) / \ln(1 + s_i), & \text{if } s_i \neq 0 \\ \frac{y_i - y_{i-1}}{t_i}, & \text{if } s_i = 0. \end{cases} \quad (4)$$

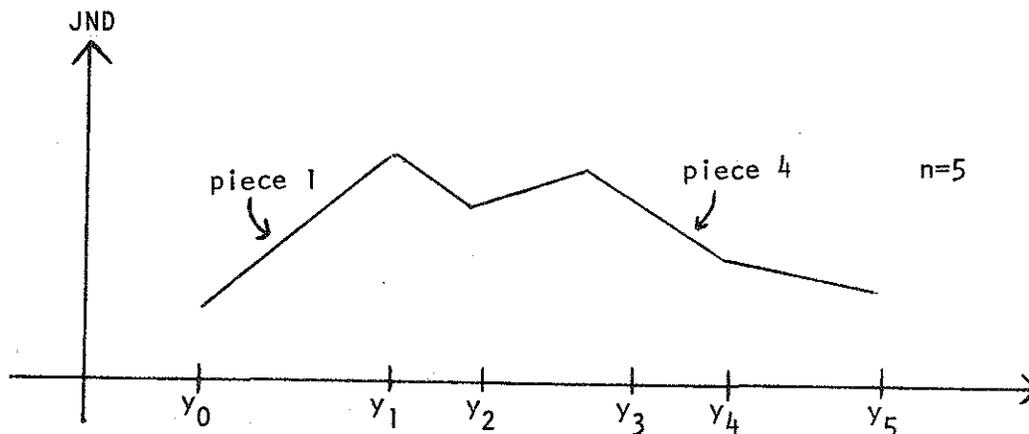


Figure 4: Piecewise linear JND curve

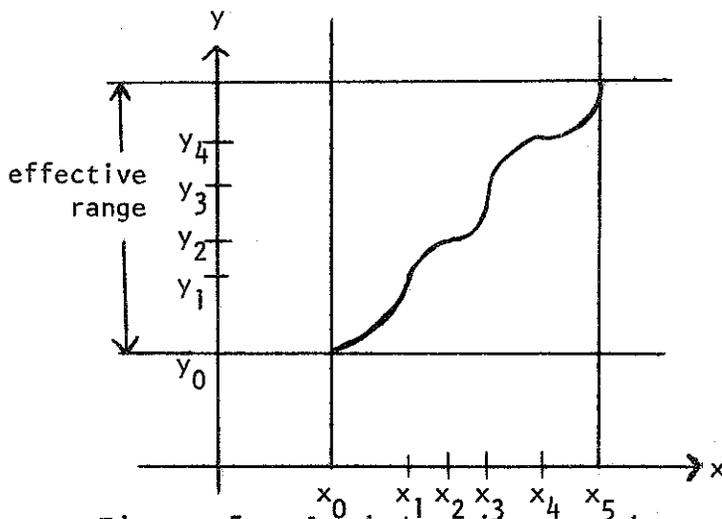


Figure 5: An intensity mapping

The objective is to find a monotonic function  $f$  from the recorded intensity  $x$ , on its range (often the effective range), to the device driving intensity  $y$ , on the effective range, such that the JND curve when considering  $x$  to be the driving intensity is constant over its range (see figure 5). This function, an intensity mapping, has no effect on the PDR. It increases contrast when the slope of  $f$  is greater than 1 and decreases it when the slope is less than 1, in such a way as to make equal changes in  $x$  equally perceivable, within its range. With this property it makes sense to speak of the change in  $x$  corresponding to 1 jnd. This value,  $\Delta x$ , is obtained by dividing the range of the recorded intensity  $x$  by the perceived dynamic range:

$$\Delta x = (x_{\max} - x_{\min}) / \text{PDR} \quad (5)$$

We must find the function  $f$  such that, for all  $j$ , integer or noninteger, between 0 and PDR,  $f(x + j \Delta x)$  is  $j$  jnd's along  $y$  from  $y_0$  according to  $\text{JND}(y)$ . Thus if we let  $x_0 = x_{\min}$  and define

$$x_k = x_0 + \left( \sum_{i=1}^k \text{PDR} \right) \Delta x, \text{ for } k=0,1,\dots,n, \quad (6)$$

then  $f$  should have the property that for  $k = 0, 1, \dots, n$ ,  $f(x_k) = y_k$ , (7)

where the  $y_k$  are the knots of the piecewise linear JND curve. Furthermore, for any  $x \in [x_{i-1}, x_i]$ , if  $x = x_{i-1} + m \Delta x$ , (8) then  $f(x)$  should be  $m$  jnd's from  $y_{i-1}$  along the  $i$ th linear piece. This point can be found as follows.

For a linear piece of a JND curve on  $[y_{i-1}, y_i]$ , moving along by  $\Delta x$  in  $x$  must correspond to moving 1 jnd in  $y$ . That is,  $f$  should have the property that for  $x \in [x_{i-1}, x_i]$ ,  $f(x + \Delta x) = f(x) + \text{JND}(f(x))$

$$= f(x) + t_i + s_i f(x) = (1 + s_i) f(x) + t_i \quad (9)$$

This difference equation for  $f(x)$ , together with the initial condition  $f(x_{i-1}) = y_{i-1}$  is solved by

$$f(x) = \begin{cases} (y_{i-1} + t_i/s_i)(1 + s_i)^{\frac{x-x_{i-1}}{\Delta x}} - t_i/s_i, & \text{if } s_i \neq 0 \\ y_{i-1} + \frac{x-x_{i-1}}{\Delta x} t_i, & \text{if } s_i = 0. \end{cases} \quad (10)$$

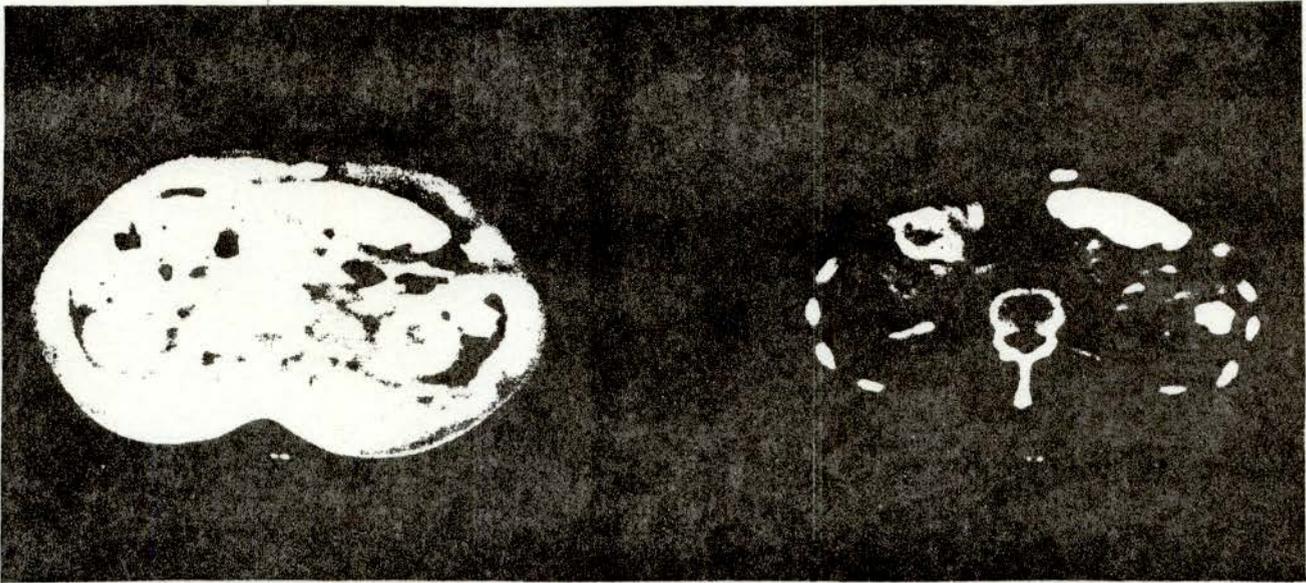
(Note that  $(x - x_{i-1})/\Delta x$  is the value denoted by  $m$  in equation 8.)

To summarize, the desired mapping is implemented as follows. We first compute the  $x_k$  once and for all by equations 3-6. For any  $x$  in the range  $[x_0, x_n]$  we find the interval  $[x_{i-1}, x_i]$  in which  $x$  falls and then apply equation 10. If the  $x$  values are to be given digitally, one can construct a look-up table of  $f(x)$  for all values of  $x$ . For each reference intensity, the corresponding tabulated value is used to drive the original display device.

The results of linearizing a display device, for two common devices, ordinary grey-scale video and an electrostatic hardcopy display using binary intensity and dithering [Jarvis et al, 1976], are given in figures 6 and 7. These results for a CT scan of the abdomen, illustrate that the difference made by linearization of a display device is nontrivial. In one of these examples it is probably the case that the image from the linearized device is better than that from the original device, and in the other example the image from the original device is probably better. The objective of the linearization is not directly to obtain an image improvement but as a required first step to allow sensible comparison and choice of imaging devices and sensible design of intensity mappings matched to the contents of a particular image and the context within which it is being viewed.

#### Acknowledgements

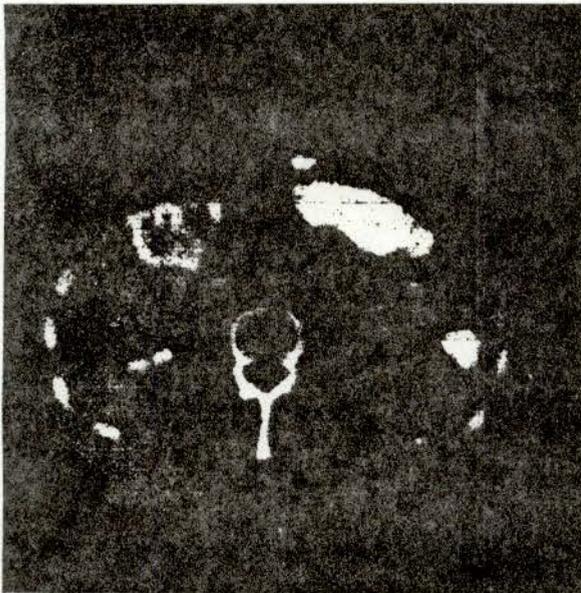
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By original device

By linearized device

Figure 6: Effect of linearization on grey-scale video display



By original device

By linearized device

Figure 7: Effect of linearization on dithered binary electrostatic plotter display

## References

Jarvis, J.F., Judice, C.N., and Ninke, W.H., "A Survey of Techniques for the Display of Continuous Tone Pictures on Bilevel Displays", Computer Graphics and Image Processing, 5 (1):1976, pp. 13-40. The ordered dither technique with  $n = 16$ , described in section 2.3, was used for figure 7.

Metz, C.E., "Basic Principles of ROC Analysis", Seminars in Nuclear Medicine, VIII (4):1978, pp. 283-298.

Pizer, S.M. and Chan, F.H., "Evaluation of the Number of Discernible Levels Produced by a Display", Traitement Des Informations en Imagerie Medicale (Information Processing in Medical Imaging), R. DiPaola and E. Kahn, eds., Editions INSERM, Paris, 1980, pp. 561-580.