# Spatio-Temporal Modeling of campus WLAN traffic demand 

Félix Hernández-Campos ${ }^{a}$ Merkourios Karaliopoulos ${ }^{b}$ Maria Papadopouli ${ }^{a, b, c}$ Haipeng Shen ${ }^{d}$<br>a. Department of Computer Science, University of North Carolina, Chapel Hill, NC, USA.<br>b. Department of Computer Science, University of Crete, Heraklion, Greece.<br>c. Institute of Computer Science, Foundation for Research and Technology - Hellas, Greece.<br>d. Department of Statistics and Op. Research, University of North Carolina, Chapel Hill, NC, USA. Emails: \{fhernand,mkaralio, maria\} @cs.unc.edu, haipeng@email.unc.edu


#### Abstract

Campus wireless LANs (WLANs) are complex systems with hundreds of access points (APs) and thousands of users. Researchers in wireless networking are faced with the challenge of constructing simulations and testbed experiments that reproduce the characteristics of these networks, and taking them into account in their theoretical work. However, there is only a limited set of modeling results in this area derived from real measurement data, and they do not provide a complete and consistent view of entire WLAN systems. In this work we propose a first system-wide, multi-level model for campus WLAN. Our emphasis is on parametric modeling, which provides a parsimonious characterization and the most flexible foundation for simulation studies. Our results are derived from large traces collected at the University of North Carolina.


## I. Introduction

Wireless networks are increasingly being deployed and the demand for wireless access grows rapidly. However, empirical and performance analysis studies indicate dramatically low performance of real-time constrained applications over wireless LANs (such as [1] on the VoIP) and clients frequently experience failures and disconnections. The wireless LANs have more vulnerabilities, bandwidth, and latency constraints than their wired counterparts. It is critical to understand the performance of the wireless networks and develop wireless networks that are more robust, easier to manage and scale, and able to utilize scarce resources more efficiently. While in several cases over-provisioning in wired networks is acceptable, it can become problematic in the wireless domain. A number of mechanisms, such as capacity planning, resource reservation, link adaptation, and load balancing, need to be employed to support such networks. To perform meaningful simulation studies and analysis of those mechanisms, the availability of models of the network and its demand is critical. Furthermore, the design of these real-life systems can take advantage of such traffic models, their temporal and spatial phenomena, and forecasting algorithms. For example, to perform load balancing among APs or resource reservation at an AP, the system needs to monitor the network demand and perform dynamically short-term forecasting. Capacity planning aims in the optimal placement of APs, channel assignments, range, VLAN configuration, and network topology. This requires the spatial modeling of the demand, and an understanding of the
evolution of the aggregate demand, its temporal characteristics, and longer-term forecasting.

Improving load-balancing, resource reservation, and capacity planning shapes our empirical measurements and modeling studies. The most intriguing aspect of such modeling is its multi-level spatial-temporal dimensions, namely, the different spatial and system scales (e.g., infrastructure-wide, AP-level or client-level) and time granularities (e.g., packet-level, flowlevel or aggregate). An important goal of this research is to model the dynamics of an entire campus wireless infrastructure and develop a methodology that characterizes the traffic demand in different levels as well as the interplay of some critical parameters.

Key elements of the demand are the client associations and flows and their parameters, namely their arrivals and sizes. We study client association dynamics using sessions, which group associations into episodes of continuous activity. The session level, captures the interaction between clients and APs, and it is fundamental for any study that deals with the state in APs (e.g., for energy conservation, load-balancing, resource reservation and allocation, and roaming). The flow-level is an important structure above the packet-level for network traffic analysis and closed-loop traffic generation. How do clients arrive at an AP or in the campus-wide infrastructure? How do flows arrive at APs? What are their temporal phenomena? Sessions and flows are interrelated: for example the load of an AP is given by the set of network flows that traverse this AP, generated by the clients associated to it. This paper models these structures in both spatial and temporal dimensions and investigates their dependencies and interplay. Finally, it uses these relations to build models for traffic load and short-term traffic forecasting (important aspects of load-balancing and resource reservation).

While there is a rich literature characterizing traffic in wired networks ([2], [3]), there are only a few studies available that examined wireless demand. The multi-level modeling of the wireless demand and its spatial and temporal phenomena has received very little attention from our community. This study builds the foundations and the methodology for measuring flow and session arrivals and their sizes at both the systemwide and AP-levels. The only closest study is the modeling of

| Component | Model | Probability Density Function (PDF) |  |
| :--- | :--- | :--- | :--- |
| Session Arrivals | Time-varying Poisson |  |  |
|  | with rate $\lambda(t)$ | $N: \#$ of sessions between $t_{1}$ and $t_{2}$ <br> $\lambda=\int_{2} \lambda(t) d t, \operatorname{Pr}(N=n)=\frac{e^{-\lambda} \lambda^{n}}{n!}, n=0,1, \ldots$ | Hourly rate: 44 (min), <br> $t_{1}$ |
| Session AP Preference | Lognormal | $p(x)=\frac{1}{\sqrt{2 \pi} x \sigma} \exp \left[-\frac{(\ln x-\mu) 2}{2 \sigma 2}\right]$ | $\mu=4.0855, \sigma=1.4408$ |
| Flow-inter-arrival/Session | Lognormal | Same as above |  |
| \# of Flows/Session | BiPareto | $p(x)=k^{\beta}(1+c)^{\beta-\alpha} x^{-(\alpha+1)}(x+k c)^{\alpha-\beta-1}$ <br> $(\beta x+\alpha k c), x \geq k$ | $\alpha=0.06, \beta=1.72$, <br> $c=284.79, k=1$ |
| Flow Size | Same as above | $\alpha=0.00, \beta=0.91$, <br> $c=5.20, k=179$ |  |

TABLE I
SUMMARY OF SYSTEM-WIDE TRAFFIC DEMAND MODEL.
traffic flows by Meng et al. [4].
The main contribution of this paper is a novel methodology for modeling the demand in large wireless networks using a system-wide, multi-level parametric approach. Our approach distinguishes two important dimensions in wireless network modeling, namely the demand (user-initiated activity through flows and sessions) and the topology (network, infrastructure, and radio propagation dependencies). This enables us to "superimpose" models for the demand on the specific topology, scaling it up and down, and focusing on the right level of detail for the performance analysis or simulation study (e.g., AP-level, system-wide, client-level). This methodology "masks" network-related dependencies that are not important for a range of systems, and make the wireless networks amenable to statistical analysis and modeling. It has been really a fascinating problem because the analysis of such large data acquired from different monitoring tools, for extended periods of times, from a very large wireless infrastructure is challenging. Furthermore, it is critical to design the right structures for modeling (e.g., sessions and flows) that are wellbehaved statistically and amenable to parametric modeling. To the best of our knowledge, this is the first system-wide multi-level modeling of the wireless networks. Currently, we are working on the combined modeling of topology, sessions and flows, developing a complete methodology for modeling wireless networks.

Besides the methodological aspects of our work, our main contribution consists of a coherent parametric model of the workload of the entire WLAN. The statistical models we propose are summarized in Table I. Our parsimonious description of the workload looks appropriate for simulation studies. Researchers can simulate the load of the network at both the client association and flow levels by simulating the compound process of sessions and flows. Sessions, which are well-defined episodes of client activity, have a well-behaved arrival process, which, as we show, can be accurately described using a time-varying Poisson process. In addition, an AP preference distribution can be used to distribute session load throughout the wireless infrastructure in a manner that is representative of real workloads. The session arrival process provides the seeds for a cluster process, in which the arrivals of sessions imply the arrivals of correlated sets of flows. Simulations can first produce an arrival process of sessions, and then sample from
the distributions of the number of flows and their inter-arrivals to produce the process of flow arrivals. Each flow is then given a size from the flow size distribution. Our main contributions are as follows:

- A novel methodology for the parametric modeling of wireless demand, in which we rely on robust statistical methods to study large scale phenomena.
- Models for flow arrivals at AP-level and system-wide (See Table I) in a more natural framework than the earlier work [4].
- Analysis of the inter-play of the session arrivals, flow arrivals, and traffic load at APs, their temporal phenomena and statistical properties (e.g., stationarity).
- A short-term forecasting algorithm at the AP-level that takes advantage of the aforementioned interdependencies.
Section II describes briefly the wireless infrastructure at UNC and the data acquisition process. Section III discusses our statistical methodologies. We describe the BiPareto distribution, and illustrate how one can use quantile plots and simulation envelopes to evaluate parametric fits. Also, we discuss a testing procedure for a time-varying Poisson process. Our modeling results are discussed in the next two sections. Section IV considers the spatio-temporal characteristics of the entire system, and how the modeling results summarized in Table I are derived. Section V applies the modeling insights developed in the system-wide analysis to the modeling of the specific APs. In Section VI, we discuss the implications of our modeling results. Section VII provides an overview of the related work. Section VIII summarizes our main results and discusses future work.


## II. Data AcQuisition

The data come from the large campus wireless network deployed at UNC. campus and a number of off-campus administrative offices. The university has 26,000 students, 3,000 faculty members, and 9,000 staff members. Personal laptops are required for undergraduates and almost all of them are equipped with a wireless interface.

The data in this paper were collected using SNMP for polling every AP on campus every five minutes. We used a custom data collection system, being careful to avoid the pitfalls described in [5]. The system was implemented using
a non-blocking SNMP library for polling each AP precisely every five minutes in an independent manner. This eliminates any extra delays due to the slow processing of SNMP polls by some of the slower APs. The UNC trace was collected between 9:09 AM, September 29th, 2004 and June 2005. The monitoring system did not suffer any problems during this period.

Most of our analysis concentrates on an 8-day period in which we also collected data about the flows in the wireless network. Our data set consists of a total of 175 GB of packet header traces collected from the link between the University of North Carolina at Chapel Hill and the rest of the Internet. The data collection took place between 12:06 PM on Wednesday April 13rd, 2005, and 22:18 PM on Wednesday, April 20th, 2005, resulting in a continuous trace of 178.2 hours. Packet headers were acquired using a high-precision monitoring card (Endace DAG 4.3 GE) attached to the receiving end of a fiber split. The card was installed in a high-end FreeBSD server. Neither the server nor the card's driver reported any failures or packet drops during the monitoring.

We do not examine datasets from other locations in this report, although we have conducted analysis of the data from Dartmouth University. In general, we find substantial similarities in the characteristics of these two WLANs, at least at the level relevant for our parametric modeling. This is in agreement with our previous work [6], which carefully compared the system-wide characteristics of UNC and Dartmouth in an exploratory manner.

## III. Statistical Methodology

Several statistical analysis tools are used in Sections IV and V for the system-wide and AP-specific modeling. We provide a description of the relevant techniques in the current section.

## A. BiPareto Distributions

The BiPareto distribution is proposed in [7] to model number of TMP connections per HTTP user session and the average inter-connection time within a session. Then, [8] shows that a family of BiPareto distributions can be used to model wireless session durations of users on a major university campus using the IEEE 802.11 wireless infrastructure.

The distribution is specified by four parameters $(\alpha, \beta, c$ and $k$ ), whose complementary cumulative distribution function (CCDF) is given by

$$
\left(\frac{x}{k}\right)^{-\alpha}\left(\frac{x / k+c}{1+c}\right)^{\alpha-\beta}, x \geq k .
$$

$k>0$ is the minimum value of a BiPareto random variable, which is a scale parameter. The CCDF initially decays as a power law with exponent $\alpha>0$. Then, in the vicinity of a breakpoint $k c$ (with $c>0$ ), the decay exponent gradually changes to $\beta>0$.

Basically, the BiPareto distribution has two Pareto tails on both ends of the distribution. On a log-log plot, a CCDF of the form $x^{-\alpha}$ (a Pareto tail) would appear as a straight line with slope $-\alpha$. Thus, the log-log plot of a BiPareto CCDF has two
nearly linear regimes, one with slope $-\left(\frac{c}{1+c} \alpha+\frac{1}{1+c} \beta\right)$ and the other one with slope $-\beta$. This is the reason that we use BiPareto distributions to model number of flows per session and flow size in Section IV. The parameters ( $\alpha, \beta, c$ and $k$ ) can be estimated via maximum likelihood [7].

## B. Quantile plots and simulation envelopes

A quantile plot is a graphical method for assessing the goodness of fit of a certain distribution to the data [9]. It plots the data quantiles versus the corresponding theoretical quantiles from the distribution being tested. The distribution parameters are estimated from the data using methods like maximum likelihood, method of moment or quantile matching. When the theoretical distribution is a good fit, the quantile plot should follow a diagonal straight line closely.

To account for possible sampling variability, a simulation envelope of 100 overlaid curves can be superimposed. Each curve is a similar quantile plot, where the "data" are simulated from the theoretical distribution. This simulation envelope provides a simple visual accounting for the sampling variability. When the theoretical distribution fits the data well, the quantile plot should lie mostly within the envelope. Several quantile plots are shown in Sections IV and V.

## C. Time-varying Poisson Processes

1) Background: Suppose $\{\Lambda(t): t \geq 0\}$ is a stochastic point process, which counts number of events (or arrivals) in $[0, t]$. Sometimes, $\{\Lambda(t)\}$ is referred to as the arrival process of the events of interest. For example, in the current paper, $\{\Lambda(t)\}$ is the arrival process of sessions to the whole wireless system or to a particular AP.
$\{\Lambda(t)\}$ is a Poisson process if it has the following two properties:
2) The number of arrivals in disjoint intervals are independent;
3) For some finite $\lambda>0$,

$$
P(\Lambda(t)=j)=e^{-\lambda t}(\lambda t)^{j} / j!, j=0,1, \ldots .
$$

Thus, for each $t, \Lambda(t)$ is a Poisson random variable with mean $\lambda t$, which is the product of the arrival rate $\lambda$ and the interval length $t$. Note that a Poisson process is a renewal process where the inter-arrival times are independent exponential[10]. It is well-known that such a process results from the following behavior: there exist many potential, statistically identical arrivals; there is a very small yet non-negligible probability for each of them arriving at any given time; and arrivals happen independently of each other. Arrival processes driven by human behaviors are usually well modeled by Poisson processes.

A closely related process is a time-varying (or inhomogeneous) Poisson process, where the arrival rate is a function of time $t$, say, $\lambda(t)$. Such a process is the result of timevarying probabilities for an event to arrive, and it is completely characterized by its arrival rate function. A smooth $\lambda(t)$ is familiar in both theory and practice in a wide variety of contexts, and seems reasonable for modeling session arrivals in Section IV-A.

Another important variation is a cluster Poisson process. Such a process starts with an underlying Poisson "seed" process. Each Poisson seed generates a random number of additional clustered points. Finally, the combined set of points are the events of the full process. To characterize this process, one needs to model the cluster size and the inter-arrival times between points within a cluster, in addition to the Poisson seed process. This process makes physical sense for many IP applications. Web pages are an excellent example, since each page consists of many embedded objects (such as graphs, banners and internal links), which need additional connections for downloading. Such a process has been used to model wired traffic in [7] and [11], and seems to be a nice candidate for modeling flow arrivals generated by sessions in Section IV-B.
2) A Statistical Test for Time-varying Poisson Processes: In this section, we describe a test [12], [13] for the null hypothesis that an arrival process is a time-varying Poisson process, with a slowly varying arrival rate.

To begin with, we break up the interval of a day into relatively short blocks of time. For convenience, blocks of equal length, $L$, are used, resulting in a total of $I$ blocks; though this equality assumption can be relaxed. For the later analysis in Section IV-A, $L$ is chosen to be 0.1 hour.

Let $T_{i j}$ denote the $j$ th ordered arrival time in the $i$ th block, $i=1, \ldots, I$. Thus $T_{i 1} \leq \ldots \leq T_{i J(i)}$, where $J(i)$ denotes the total number of arrivals in the $i$ th block. Define $T_{i 0}=0$ and

$$
\begin{equation*}
R_{i j}=(J(i)+1-j) \ln \left(\frac{L-T_{i, j-1}}{L-T_{i j}}\right), \quad j=1, \ldots, J(i) \tag{1}
\end{equation*}
$$

Under the null hypothesis that the arrival rate is constant within each time interval, the $\left\{R_{i j}\right\}$ will be independent standard exponential variables as we now discuss.

Let $U_{i j}$ denote the $j$ th (unordered) arrival time in the $i$ th block. Then the assumed constant Poisson arrival rate within this block implies that, conditioning on $J(i)$, the unordered arrival times are independent and uniformly distributed between 0 and $L$. Denote $V_{i j}=\frac{L}{L-U_{i j}}$, and it follows that $V_{i j}$ are independent standard exponential. Note that $T_{i j}=U_{i(j)}$, thus

$$
V_{i(j)}=\ln \left(\frac{L}{L-U_{i(j)}}\right)=\ln \left(\frac{L}{L-T_{i j}}\right) .
$$

As one can see, $R_{i j}=(J(i)+1-j)\left(V_{i(j)}-V_{i(j-1)}\right)$. Then, the exponentiality of $R_{i j}$ follows from the following wellknown lemma.

Lemma: Suppose $X_{1}, \ldots, X_{n}$ are independent standard exponential, then $Y_{i}=(n-i+1)\left[X_{(i)}-X_{(i-1)}\right], i=2, \ldots, n$, are independent standard exponential.

Any customary test for the exponential distribution can then be applied to $R_{i j}$ for testing the null hypothesis. For example, the familiar Kolmogorov-Smirnov test or Anderson-Darling test [14] could be used. These nonparametric tests are based on deviations between the empirical cumulative distribution function (CDF) of the data and the hypothesized theoretical CDF. However, as noted in [15], statistical significance tests are not very useful when facing large data sets, because they always give insignificant results no matter what. Thus, we
prefer to test the exponentiality using a graphical tool, such as an exponential quantile plot with a simulation envelope as described in Section III-B.

## IV. System-Wide Modeling

The workload of a wireless network is created by clients that access the infrastructure to communicate with other Internet hosts. At the most basic level, APs are in charge of forwarding IP packets, providing a bridging service between the wireless medium and the wired network. At a higher level, APs are also in charge of client dynamics, allowing clients to associate and disassociate from the wireless network, and implementing transparent roaming, which enables a client to move from one AP to another while maintaining connectivity. From the modeling perspective, this creates two levels at which the workload of the wireless infrastructure can be studied: the packet forwarding level and the client association level. These two levels are not independent of each other. Wireless clients can only use the packet forwarding mechanism when they are associated to an AP, and problems with the association level can easily result in the loss of the client's connectivity. In this paper, we consider the problem of modeling these two levels jointly, in a manner that can support more comprehensive and flexible simulations and testbed experiments. This is a formidable modeling challenge, since we focus on a large wireless network with hundreds of APs and thousands of clients. It is easy to study this problem from many different points of view, as the growing literature on wireless network measurement highlights [16], [17], [18], [19], [5], [4]. Our goal is, however, to create a first solution to this modeling problem, reducing it to a basic set of characteristics that are amenable to parametric modeling. One of our contributions is methodological, in the sense that we propose a reduction of the modeling to some essential components that could easily be enriched in many different ways.

Our modeling is based on two fundamental concepts, a wireless session and a network flow. A wireless session can be loosely defined as a separate episode in the interaction of a client and the wireless infrastructure. The most basic example is a wireless client that arrives at the network, associates to one AP for some period of time, and then leaves the network. A single session can also include several associations, as long as they occur close in time, and visits to several different APs. The crucial observation is that sessions provide a natural toplevel for the modeling of wireless network workloads. As we will demonstrate, sessions are statistically well-behaved, which makes it possible to construct a parsimonious description of the system. The concept of session is robust to networkdependencies. As Paxson and Floyd argued, in the context of their traffic modeling work[20], the most flexible and representative type of modeling should not incorporate network characteristics that are too specific to the network conditions in the data from which the model is derived. Otherwise, simulations and experiments that use this model can never study changes in those conditions or new network mechanisms that shape those conditions. For example, modeling the precise
sequence of associations and disassociations inside sessions is too network-specific, since small changes (e.g., in the network topology, environment, range of the equipment), can dramatically change association/disassociation dynamics. If a researcher wants to study a new and more robust algorithm for AP selection, this new algorithm will also change association dynamics, so the simulation should not impose an arbitrary sequence of associations and disassociations. In this regard, a session, as the unit of continuous use of the infrastructure by a wireless client, can make simulations more representative. The simulated session may end up having completely different association dynamics, but the essence of the workload it represents, a client utilizing the network for some period of time, is preserved.

Besides associations dynamics, a session also represents a unit of load at the packet forwarding level. A session includes all the packets sent and received by the APs due to the client's communication with one or more Internet hosts. As demonstrated in [4], and again in agreement with the principles of network-independent modeling from [20], the right way of modeling the packet forwarding workload is to examine network flows. Network flows, such as TCP connections and UDP conversations, are well-separated collections of packets between a pair of Internet hosts, i.e., packets that share the same transport-layer " 5 -tuple". In our model, a session groups the set of flows started by a client. Simulating the system therefore consists of simulating sessions and the flows that are started inside them, leaving the actual packet-level (and association) simulation to underlying mechanisms. These other mechanisms are independent of our model.

The rest of this section presents our modeling as applied to the characterization of the entire wireless network. We first discuss the process of session arrivals, in Section IV-A, which is the starting point of the entire approach. We then consider joint modeling of sessions and flows in Section IVB, where sessions are seen as seeds for the arrivals of groups of flows. Finally, we consider the sizes of these flows and their impact on the infrastructure in Section IV-C. This system-wide characterization, by the virtue of the substantial aggregation, makes the statistical modeling more tractable. In Section V, we also examine the modeling of individual APs, and how findings from the system-wide model apply to the AP-specific modeling.

## A. Session Arrivals

The starting point of our model is the process of session arrivals. Figure 1 shows the point process of session arrivals of an 8-day trace. Each dot in the scatterplot corresponds to the arrival of a session, and each arrival is placed according to its temporal coordinates (arrival time in x -axis) and its spatial coordinates (arrival AP in the y-axis). Session arrivals vary widely, but some patterns are apparent. First, there is a clear periodicity which is caused by the substantial decrease of activity in the network during the nights. Another temporal characteristic of session arrivals is the decrease of activity during the weekends (days 3 and 4 in the plot). Figure 2


Fig. 1. Arrivals of sessions from wireless clients over time and across the campus APs.


Fig. 2. Time-series of session arrivals in the entire campus WLAN (1-hour bins).


Fig. 3. Probability that a session is started in a specific AP, which we call the AP preference distribution.
provides an even clearer picture of these diurnal/nocturnal and weekday/weekend periodicities. The plot shows the timeseries of session arrivals for the entire system using 1-hour bins. The time-series plot shows sharp increase in the number of session arrivals in the morning, reaching a peak between 1,000 and 1,110 sessions per hour during weekdays and 350 session arrivals per hour during weekends. This pattern holds across our entire dataset, which includes ten months of session arrivals, although specific events, such as the Christmas break, decreased the activity considerably.

In terms of the spatial characteristic of the session arrivals process, Figure 1 provides a first overview of the way sessions arrive to specific APs in the infrastructure. Our mapping of the APs to location in the $y$-axis is random, but it clearly shows the wide spatial variability of the workload. The temporal patterns appear throughout the infrastructure, although some APs seem more likely to be used at night than others. Figure 3 shows the probability that a session is started at a given AP. Note that the numbering is not preserved from Figure 1, since APs in this plot are sorted from left to right by decreasing popularity as a session starting point. The plot shows that a few top APs receive a substantial fraction of all sessions, e.g., almost $20 \%$ for the most common starting AP. It also shows a nonnegligible tail, so most APs are starting points of wireless sessions.
One remarkable aspect of Figures 2 and 3 is the smoothness of the curves, which suggests phenomena that are amenable to modeling. Our analysis reveals that session arrivals follow a time-varying Poisson process, and that AP preference is accurately described by a lognormal distribution. We model the session arrival process using the time-varying Poisson model described in Section III-C. In order for this model to be valid, the $R_{i j}$ s as defined in (1) during short time blocks must be exponentially distributed with a parameter of 1 , and uncorrelated. The top part of Figure 4 shows an exponential


Fig. 4. The $R_{i j}$ s are independent and exponentially distributed. Only one hourly block is shown here, but the results are consistent across the entire dataset.


Fig. 5. Lognormal model of AP preference distribution.
quantile plot of the $R_{i j}$ s during one randomly chosen hour. We set $L=0.1$ hour in calculating the $R_{i j} \mathrm{~s}$. The red quantile plot follows closely the green diagonal line, and remains well within the blue simulation envelope. This suggests that the exponential fit is clearly appropriate. The maximum likelihood estimate of the exponential parameter is 0.9372 , which is very close to 1 , and agrees with the claim that the $R_{i j} \mathrm{~s}$ are standard exponential. The bottom plot of the figure plots the autocorrelations of the $R_{i j} \mathrm{~s}$ up to 20 lags. The sample autocorrelations are always within the confidence intervals, so the $R_{i j} \mathrm{~s}$ do not exhibit any significant correlations. We conduct the same analysis for all the 192 hours of the 8 -day dataset considered in this section, and the results are similar.
Our analysis in Figure 5 shows that a lognormal distribution with parameters $\mu=4.0855$ and $\sigma=1.4408$ is a good model for the distribution of AP preference. As we can see, the original data, shown in red, are within the natural variability of the lognormal model, since it remains within the blue simulation envelope. The only departure from lognormality is for the smallest values, i.e., the most unlikely starting APs, where the number of samples is very small. Overall, the lognormal distribution is an excellent description of the data. We have also considered other models but they are clearly outperformed by the lognormal fit. For example, Zipf's law, a classic way of describing popularity, is very far from the AP preference distribution in our data. Our AP preference model characterizes the spatial allocation of session arrivals in the sense that it captures the way sessions are distributed throughout the infrastructure. It does not capture AP coordinates in space, which are specific to the infrastructure. This is a difficult problem, which has to deal with a 3D environment (latitude, longitude and height), and perhaps with the layout of the environment (which has high impact on radio coverage). The ideas from the area of statistical analysis of spatial point patterns can be helpful here, but we do not present our efforts in this direction in this report.


Fig. 6. Number of flows per session.


Fig. 7. Stationarity of the distribution of the number of flows per session (body).

## B. Cluster Poisson Model of Flow Arrivals

Below the association level, each session consists of a set of flows that represent the communication between a wireless client and one or more Internet hosts. In this view, the arrival of a session represents the correlated arrival of a group of flows. It is therefore natural to describe flow arrivals as a cluster process rather than a point process in which flows arrivals are described in isolation. Our model considers that the arrival of a cluster of flows is triggered by the arrival of a session, which is seen as the seed of the cluster process. Modeling this process requires to describe the arrival process of sessions, which is presented above in Section IV-A, the number of flows associated to each session, i.e., to each cluster, and the interarrivals of flows within sessions. Given that the arrival process of sessions is a (time-varying) Poisson process, we say that the process of flow arrivals is a cluster Poisson process. There are well developed methods for simulating time-varying Poisson processes, for example, the thinning method described in [21], [22]. Along with models for session sizes, we can generate synthetic traffic traces.

Our analysis of the distribution of the number of flows per session reveals that the most appropriate parametric model is


Fig. 8. Stationarity of the distribution of the number of flows per session (tail).
the BiPareto distribution described in Section III-A. Figure 6 shows the fit of this distribution to our empirical data using a $\log -\log$ plot of the CCDF, i.e., $\log _{10}(\operatorname{Pr}\{X>x\})$ vs. $\log _{10} x$. The red circles are an equidistant set of samples from a BiPareto distribution with parameters $\alpha=0.06, \beta=$ $1.72, c=284.79$ and $k=1$. These circles are right on top of the empirical distribution of the number of flows (in blue) for probabilities between 0 and 0.995 , i.e., $99.5 \%$ of the distribution. The fit is worse for the remaining $0.5 \%$, but this is already in a region of the tail that is very variable due to sampling artifact. In any event it is clear that the BiPareto model fits the empirical distribution very well.

We have also studied the stationarity of the distribution of the number of flows per session. Figures 7 and 8 show one empirical distribution for each of the 8 days in the dataset, demonstrating striking consistency. This is a strong indication of the feasibility of modeling the system using parametric models. Figure 7 shows the bodies of the distributions of the number of flows per session using a log-log plot of the CDF, i.e., $\log _{10}(\operatorname{Pr}\{X \leq x\})$ vs. $\log _{10} x$. The eight distributions are very similar, with the vast majority of the sessions featuring between 1 and 1000 flows. The distributions for the weekends are slightly heavier. Figure 8 shows the tails of the distributions using CCDFs, again showing similar shapes. The number of flows per session goes as far as 10,000 for $0.1 \%$ of the sessions.

The second component of our cluster model is the distribution of the flow inter-arrivals within sessions. We show that a lognormal model provides good fit, although the distribution is rather complex. Figure 9 shows the lognormal quantile plot for the empirical data, and the parameters are estimated to be $\mu=-1.3674$ and $\sigma=2.785$ using maximum likelihood. The red quantile plot follows the green diagonal line closely for all of the quantiles. The simulation envelope is very narrow in this case, and shows that some deviations from the lognormal model in the upper part are significant. While more complex models may provide a better approximation, i.e., an ON/OFF model, our lognormal fit certainly provides a reasonable description of the data using only two parameters.


Fig. 9. Flow inter-arrivals: lognormal quantile plot of the data with a simulation envelope.


Fig. 10. Stationarity of the distribution of flow inter-arrivals within sessions (body).


Fig. 11. Stationarity of the distribution of flow inter-arrivals within sessions (tail).


Fig. 12. Average inter-arrival across sessions.


Fig. 13. BiPareto Model of Flow Sizes.

As in the case of the distribution of the number of flows per session, we have also studied the stationarity of the distributions of the flow inter-arrivals within sessions. Figures 10 and 11 show that the flow inter-arrivals are very consistent when we compare the 8 days in the dataset.
To completely understand flow arrivals, one needs to investigate the correlation structure among the inter-arrivals as well. The earlier flow modeling paper [4] shows that flow inter-arrivals across all sessions are distributed according to a Weibull model, but does not investigate the correlation problem. We plan to investigate the correlation issue in future.

## C. Flow Sizes and System Load

To capture the load of the system at the packet forwarding level in a manner suitable for closed-loop simulation and testbed experiments, it requires to describe not only the way flow arrives, but also their sizes in terms of the number of bytes that they carry. Flow arrivals can be modeled using the cluster Poisson model as established in Section IV-B. Our statistical analysis reveals that flow sizes can be accurately described using a BiPareto distribution with parameters $\alpha=0.00, \beta=$ $0.91, c=5.20$ and $k=179$. Figure 13 shows the BiPareto


Fig. 14. Stationarity of the distributions of flows sizes (tail).


Fig. 15. Sizes of sessions and sizes of flows.
fit (red circles) to the empirical data (blue curve). The fit is excellent for most of the distribution, and the BiPareto cleanly captures the transition in the slope between the body and the heavy tail of the empirical distribution. The approximation appears heavier than the empirical data at the end of the tail, which could motivate further refinements of the fit. A more complex model, such as the double-Pareto lognormal in [23], could certainly provide a closer fit, but the proposed BiPareto provides a reasonable parsimonious description.
Figure 14 examines the stationarity of the distribution of flow sizes. The distributions for 8 different days have very consistent tails, so our model seems widely applicable. Figure 15 compares the distributions of session and flow sizes. The size of a session is the sum of the sizes of its flows. The distribution of session sizes is far heavier than the distribution of flow sizes. This further reinforces the need for modeling the clustering of flows into sessions, since the combined impact of flows with correlated arrivals can stress the wireless network far more than uncorrelated flows.

## V. AP-Specific Modeling

Our joint modeling of the wireless LAN at session and flow levels can also be applied to individual APs. Intuitively,


Fig. 16. The $R_{i j}$ s in AP 222 are independent and exponentially distributed. One randomly chosen hour is shown.
looking at single APs is more difficult, since the reduction in the level of aggregation makes the data less well-behaved. However, we will demonstrate that the modeling insights from the system-wide modeling in Section IV are also useful here. We focus on AP 222, one of the hotspots of UNC's wireless network. The parameters derived from our modeling of AP 222 are shown in Table II.

Section IV-A shows that the process of session arrivals to the entire system can be described using a time-varying Poisson process. This is also the case for the process of session arrivals to AP 222. Similar to Section IV-A, we random select one hour during which there are more than 10 session arrivals to AP 222, and divide it into ten 6-minute blocks and calculate the $R_{i j} \mathrm{~s}$ according to (1). The top part of Figure 16 shows an exponential quantile plot of the $R_{i j} \mathrm{~s}$, which suggests that the exponential fit is clearly appropriate. The maximum likelihood estimate of the exponential parameter is 0.9027 , which is very close to 1 . The bottom plot of the figure plots the autocorrelations of the $R_{i j}$ s up to 20 lags, from which one can tell that there is no much correlation among the $R_{i j} \mathrm{~s}$. We obtain similar results for all the hours during the 8 -day trace, which have at least 10 arrivals. The threshold of 10 arrivals is chosen rather subjectively to ensure a large enough sample for the quantile plots.

The finding of the Poisson session arrival process at AP 222 empirically supports our notion of the AP preference function shown in Figure 3. It is well known that if a Poisson process is randomly partitioned into several point processes according to a set of fixed probabilities, then the resulting point processes are still Poisson processes, and the rates are proportional to the respective partition probabilities. In our study, the AP preference probabilities work as the partition probabilities. As a result, the session arrival processes to separate APs should be approximately Poisson. This observation also suggests one algorithm to allocate session arrivals to the system to specific APs. After one simulates a certain number of sessions for the


Fig. 17. BiPareto model of number of flows per session in AP 222.


Fig. 18. Simulation envelope for BiPareto fit of flows per session in AP 222.
whole network, one can allocate them to different APs using their corresponding AP preference probabilities.

When we consider a single AP, the number of flows per session can also be described with great accuracy using a BiPareto distribution, as demonstrated in Figure 17. A BiPareto simulation envelope is superimposed in Figure 18, which shows that the fit is clearly excellent, even for the values with the smallest probability located in the far part of the tail. We next studied the flow inter-arrivals within the sessions that started in AP 222, and the lognormal model proposed for the entire system remains applicable here. Figure 19 shows the corresponding lognormal quantile plot. The two parameters are estimated to be -1.6355 and 2.6286 using maximum likelihood. Although the fit is worse than the one for the system-wide modeling, the quantile plot again follows the diagonal line closely, and the fit could still be useful.

Figures 20 and 21 consider BiPareto models for the sizes of flows and sessions that started from AP 222. In both cases, the BiPareto fits are excellent. Note that the session size distribution has a much heavier body than the distribution of flow sizes, but the maximum values are of similar magnitudes in both tails. Using different traces, [8] show that session


Fig. 19. Flow inter-arrivals in AP 222 are well-modeled by a lognormal distribution.


Fig. 20. Model of flow size for AP 222.


Fig. 21. Model of session size for AP 222.
durations can be modelled using BiPareto distributions as well. Our current model for session sizes complements the duration model in [8] nicely.

## VI. DISCUSSION

## A. Applying models for the AP-level demand on forecasting

Understanding the hourly traffic load characteristics at the AP-level is important for load-balancing and resource reservation. If the AP can estimate its traffic for the next hour, it can employ load balancing algorithms among neighboring APs, advise clients for its traffic load and enhance the APselection process, and notify in case of abnormal patterns of demand. Forecasting traffic load in wireless network has received very little attention, and in [24], we analyzed some simple forecasting algorithms based on recent history, diurnal pattern, and week-of-day periodicity. The hourly traffic load at an AP is quite bursty and simple forecasting algorithms performed poorly. Motivated by the strong correlation in the $\log -\log$ scale of the number of active flows and traffic load, we designed some new traffic demand algorithms based on the number of active flows.

For each TCP flow ( $j$ ), we maintain the following information, its starting time $(s(j))$, that indicates the specific second the flow was initiated, its duration $(d(j))$, and its total amount of bytes exchanged $(f(j))$ between the wireless client and the AP during that specific time. Based on this information, we create for each AP (e.g., AP $i$ ), two time series $T_{i}(t)$ and $A_{i}(t)$ in an hourly basis. $T_{i}(t)$ corresponds to the aggregate traffic of all active flows during that hour $t$ in AP $i$ and $A_{i}(t)$ to the total number of such active flows. For computing the $T_{i}(t)$, we assumed constant-bit-rate flow during the period the flow was active and aggregated over all flows (i.e., $\left.T_{i}(t)=\sum_{\forall j, s(j) \in[t, t+1)} f(j) *(t+1-s(j)) / d(j)\right)$.

We employed the following simple hourly traffic model for each AP (e.g., AP $i$ ). The predicted traffic at AP $i$ at the $t$-th hour will be $F_{i}(t) . \log F_{i}(t)=a+b * \log A_{i}(t-1)$, where $a$ and $b$ are the resulting weights of multiple regression applied in the hourly traffic of $\mathrm{AP} i, T_{i}(t)$ and number of active flows $A_{i}(t)$.

We identify the hotspot APs (most over-utilized APs based on their maximum hourly, daily, and total traffic as defined in [24]) and used the aforementioned forecasting model to predict the next-hour traffic. Specifically, the forecasting algorithm for each hotspot AP (e.g., $i$ ) looks up the number of active flows at the previous hour $\left(A_{i}(t-1)\right)$ and forecasts the traffic for the current hour $F_{i}(t)$.

To evaluate the performance of the prediction algorithms, we compute the prediction error ratio which is the ratio of the absolute difference of the predicted from the actual traffic over the actual traffic. We do not consider the entries in which the actual traffic is equal to zero. A perfect prediction algorithm has prediction error ratio equal to 0 . For each AP, we compute its mean and median ratio, $\bar{e}_{i}$ and $\hat{e}_{i}$, respectively. Using this very simple forecasting algorithm, we were able to reduce the average mean ratio $\bar{e}$ for all hotspot APs significantly. Specifically, the mean $\bar{e}$ using the new forecasting algorithm
is 9 (with a median $\hat{e}$ equal to 0.89 ) as opposed to 185 (with a median $\hat{e}$ equal to 0.67 ) in [24].

We would like to note that the set of hotspots in each trace is different. The current analysis considers only a very limited (3-day) history (Thursday, Friday, and Monday) as opposed to a five-week period in the earlier work.

The aforementioned forecasting model ignores the temporal characteristics of the flows. A next step is to use a larger tracing period, extend the model with additional flow-related information, such as its diurnal patterns, port numbers, and client profile. Furthermore, it would be interesting to explore traffic forecasting not only at the AP-level but also at the client level, and at different time-scales.

## B. Mobility

Although in the campus-wide wireless network most of the inter-AP transitions are triggered by transient changes in the environment (e.g., obstacles, density of people around) and not necessarily by user movement, there is still user mobility. For example, we found sequences of continuous (i.e., without disconnection from the wireless infrastructure) AP transitions that belong to buildings in a relative large geographic range that can only be explained by actual user mobility [8]. This mobility analysis was carried out for both sessions and clients and identified the percentage of mobile sessions for each client. Furthermore, in [25], we modeled the transitions of a client as a Markov-chain based on its history, and in [8], the visit duration at an AP as a BiPareto distribution.

The session and flow structures allow us to separate nicely the traffic demand from the network-topology dependencies and radio-propagation effects. Specifically, they give us the flexibility to superimpose the session and flow models on the infrastructure models and simulate wireless networks. Such two-dimension models are very important because they can provide a more complete picture of the network, and at the same time, all the important components to scale-up or down the network, and focus on the required level for a given performance analysis or simulation study.

We are currently working on modeling the infrastructure as a graph in which an AP corresponds to a node in the graph, and an edge between two nodes is created depending on the interAP transitions and characteristics of the APs (e.g., location, range, channel). Part of this effort is to identify different characteristics of this graph (e.g., degree of connectivity, in/out bound edges, connected components).

We believe that given such an infrastructure model, the distributions of sessions to clients, an AP-preference, and a distribution of visits to APs, we have all the important building blocks to simulate the mobility in a wireless infrastructure: We can use the AP-preference (proposed in this paper) to distribute the sessions across the infrastructure model, the visit and session durations ([8]), and the Markov-chain model for the transitions of a client ([25]) in combination with the infrastructure model.

| Component | Model | Parameters |
| :--- | :--- | :--- |
| Session Arrivals | Time-varying Poisson with rate $\lambda(t)$ | Hourly rate: 1 (min), 928 (max), 11 (median) |
| Flow-inter-arrival/Session | Lognormal | $\mu=-1.6355, \sigma=2.6286$ |
| \# of Flows/Session | BiPareto | $\alpha=0.07, \beta=1.75$, |
|  |  | $c=295.38, k=1$ |
| Flow Size | BiPareto | $\alpha=0.00, \beta=1.02$, |
|  |  | $c=15.56, k=111$ |

TABLE II
SUMMARY OF OUR AP-SPECIFIC MODEL (AP 222).

## VII. Related Work

Balazinska and Castro [17] used SNMP to characterize the WLAN in three IBM buildings ( 177 APs). The study examined the maximum number of simultaneous users per AP (mostly between 5 and 15), total load and throughput distributions. Two interesting observations made in this paper are that offered load and number of users are weakly correlated, and that user transfer rates are dependent on the location of the AP. Balachandran et al. [18] performed measurements in a three-day conference setting, also focusing on the offered network load and global AP utilization. They characterized wireless users and their workload and addressed the network capacity planning problem. The overall bursty behavior and peaks and troughs are similar at all APs, though the absolute peak throughput at each AP varies. They observed that offered load is more sensitive to individual client traffic characteristics rather than just the total number of clients.

Kotz et al. [19], [5] studied the WLAN at Dartmouth College using syslog, SNMP, and tcpdump traces. Their first study [19] reported the distribution of average daily traffic for 451 APs, which ranged from 39 MB to more than 2 GB , and observed that maximum daily traffic was far larger than the average daily traffic. In their follow-up study [5], they reported the average number of active cards per active AP per day (23 in 2001, and 6-7 in 2003/2004), and average daily traffic per AP by category (2-3 times higher in 2003/2004; twice or thrice more inbound than outbound traffic). A subset of the same data (syslog messages and tcpdump traces from 31 APs in 5 buildings) was revisited by Meng et al. [4] for flow modeling purposes. The authors proposed a two-tier (Weibull regression) model for the arrival of flows at APs and a Weibull model for flow residing times, and they also observed high spatial similarity within the same building. The authors also study the modeling of flow size, and suggest that a log-normal model provides the best approximation. This is consistent with the large body of work on this topic for wired networks and file systems (e.g., [26], [23], [27]).

The goal of our work is to bridge the results from the flow modeling in [4] and earlier exploratory work in a more comprehensive framework that takes into account the different levels at which the WLAN operates. We also tackle the lack of analysis and modeling of flow arrival dependencies in [4], using the compound process ideas from [7].

In an earlier research effort, using syslog traces, we distinguished wireless clients based on their inter-building mobility, their visits to APs, their continuous walks in the wireless
infrastructure, and their wireless information access during these periods. The user association patterns can be modeled based on the sequence of APs and their visit duration at each AP [8]. Such sequence of associations to APs can be modelled with a Markov chain. For each client, based on this model, we can predict with high probability ( $86 \%$ ) the AP with which it will get associated [25].

Also, we showed that time-varying Poisson processes can model well the arrival processes of clients at APs. These results were validated by modeling the visit arrivals at different time intervals and APs. Furthermore, there is a clustering of APs based on their visit arrival and functionality of the area in which these APs are located [13].

Using snmp-based traces, we characterized the traffic load of APs and found that both the total traffic load and number of associations at each AP follow a lognormal distribution. The logarithms of the total traffic load and total number of associations at each AP are strongly correlated. There is also a dichotomy of APs: there are APs with the majority of their clients to be uploaders and APs in which the majority of clients are downloaders [6].
Finally, in [25], we analyzed wireless web-traces to investigate the benefits of different caching paradigms in wireless networks, namely, the user-local cache, cache attached to APs, campus-wide caches, and peer-to-peer caching.

## VIII. Conclusions and Future Work

This paper introduces a novel methodology for modeling the wireless access and traffic demand by providing a multilevel perspective: it models the arrival and size of sessions and flows system-wide and at AP-level. It investigates their statistical properties, dependencies and inter-relations. It shows the stationarity of the number of flows and flow inter-arrival in a session.

In the wireless community, most of the modeling effort has been on the AP-level. The shift to sessions and flows has gained two important advantages: Sessions as opposed to visits at an AP can mask the network-related dependencies that are not important in a range of applications and simulation environments (e.g., brief transitions from one AP to another due to a transient behavior of the signal) and exhibit nice statistical properties (such as stationarity) that makes them amenable to modeling.
A further refinement of our model will consider how the size of the population of wireless users relates to the process of session arrivals. Clients are difficult to understand, due to the wide range of behavior and pervasive non-stationarities.

Some clients use the infrastructure only one or a few times, and then disappear from the system, while others represent a more constant load. Understanding this part of the workload will make simulations more intuitive, in the sense that the input could be the number of clients and perhaps some parametric description of their long-term access patterns. It is sometimes desirable to rely on some concept of the number of users in the system rather than the more abstract rate of session arrivals.

We will further explore the spatial distribution of the flows and sessions in the network in various scales. Such spatial models could be very beneficial in simulating different sizes of wireless networks (for scaling up or down a network) and studying their spatial evolution.

We are in the process of applying the proposed methodology on wireless traces acquired from very diverse infrastructures (e.g., institute-wide, technological and research park, metropolitan area and municipalities networks) to validate and enrich our models.

## ACKNOWLEDGMENT

This work was partially supported by the IBM Corporation under an IBM Faculty Award 2004/2005 grant.

## REFERENCES

[1] F. Anjum, M. Elaoud, D. Famolari, A. Ghosh, R. Vaidyanathan, A. Dutta, P. Agrawa, T. Kodama, and Y. Katsube, "Voice performance in WLAN networks -An experimental study," in Proc. of IEEE Conference on Global Communications (GLOBECOM), San Francisco, CA, USA, Dec. 2003.
[2] W. Willinger, M. S. Taqqu, R. Sherman, and D. V. Wilson, "Selfsimilarity through high-variability: Statistical analysis of Ethernet LAN traffic at the source level," $A C M C C R$, vol. 25, no. 4, pp. 100-113, Oct. 1995.
[3] M. Crovella and A. Bestavros, "Self-similarity in world wide web traffic: Evidence and possible causes," in Proc. of ACM SIGMETRICS, Philadelphia, PA, USA, May 1996.
[4] X. G. Meng, S. H. Y. Wong, Y. Yuan, and S. Lu, "Characterizing flows in large wireless data networks," in Proc. of ACM MobiCom, Philadelphia, PA, USA, Oct. 2004, pp. 174-186.
[5] T. Henderson, D. Kotz, and I. Abyzov, "The changing usage of a mature campus-wide wireless network," in Proc. of ACM MobiCom, Philadelphia, Sept. 2004.
[6] F. Hernandez-Campos and M. Papadopouli, "A comparative measurement study of the workload of wireless access points in campus networks," in in Proc. of 16th Annual IEEE International Symposium on Personal Indoor and Mobile Radio Communications, Berlin, Germany, Sept. 2005.
[7] C. Nuzman, I. Saniee, W. Sweldens, and A. Weiss, "A compound model for TCP connection arrivals for LAN and WAN applications," Computer Networks, vol. 40, no. 3, pp. 319-337, 2002.
[8] M. Papadopouli, H. Shen, and M. Spanakis, "Characterizing the duration and association patterns of wireless access in a campus," in Proc. of 11th European Wireless Conference, Nicosia, Cyprus, Apr. 2005.
[9] J. S. Marron, F. Hernandez-Campos, and F. D. Smith, "A sizer analysis of IP flow start times," Institute of Mathematical Statistics Lecture Notes - Monograph Series, J. Rojo and V. Perez-Abreu (Eds), vol. 44, pp. 87105, 2004.
[10] S. T. Ross, Stochastic Processes. John Wiley \& Sons, New York, 1995.
[11] C. Park, H. Shen, F. Hernandez-Campos, J. S. Marron, and D. Veitch, "Capturing the elusive poissonity in web traffic," in Proc. of 16th IEEE/ACM International Symposium on Modeling, Analysis, and Simulation of Computer and Telecommunication Systems (MASCOTS), Monterey, CA, USA, 2006.
[12] L. D. Brown, N. Gans, A. Mandelbaum, A. Sakov, H. Shen, S. Zeltyn, and L. Zhao, "Statistical analysis of a telephone call center: a queueingscience perspective," Journal of the American Statistical Association, vol. 100, pp. 36-50, 2005.
[13] M. Papadopouli, H. Shen, and M. Spanakis, "Modeling client arrivals at access points in wireless campus-wide networks," in Proc. of 14th IEEE Workshop on Local and Metropolitan Area Networks, Chania, Crete, Greece, 2005.
[14] R. B. D'Agostino and M. A. Stephens, Goodness-of-Fit Techniques. Marcel Dekker, 1986.
[15] P. Barford and M. E. Crovella, "Generating representative Web workloads for network and server performance evaluation," in Proc. of ACM SIGMETRICS, July 1998, pp. 151-160. [Online]. Available: http://www.cs.bu.edu/faculty/crovella/paper-archive/sigm98-surge.ps
[16] D. Tang and M. Baker, "Analysis of a local-area wireless network," in Proc. of ACM MobiCom, Boston, Aug. 2000, pp. 1-10.
[17] M. Balazinska and P. Castro, "Characterizing mobility and network usage in a corporate wireless local-area network," in Proc. of MobiSys, May 2003. [Online]. Available: http://nms.lcs.mit.edu/ mbalazin/wireless/wireless-mobisys03.pdf
[18] A. Balachandran, G. Voelker, P. Bahl, and V. Rangan, "Characterizing user behavior and network performance in a public wireless LAN," in Proc. of ACM SIGMETRICS, June 2002.
[19] D. Kotz and K. Essien, "Analysis of a campus-wide wireless network," Dept. of Computer Science, Dartmouth College, Tech. Rep. TR2002-432, September 2002. [Online]. Available: http://www.cs.dartmouth.edu/reports/abstracts/TR2002-432/
[20] V. Paxson and S. Floyd, "Wide-area traffic: the failure of Poisson modeling," in Proc. of ACM SIGCOMM, London, United Kingdom, Aug. 1994, pp. 257-268.
[21] P. Lewis and G. Shedler, "Simulation of nonhomogeneous poisson process by thinning," Naval Research Logistics Quarterly, vol. 26, pp. 403-413, 1979.
[22] K. P. White, "Simulating a nonstationary poisson process using bivariate thinning: the case of "typical weekday" arrivals at a consumer electronics store," vol. 1, 1999, pp. 458-461.
[23] F. Hernandez-Campos, J. S. Marron, G. Samorodnitsky, and F. D. Smith, "Variable heavy tails in internet traffic." Performance Evaluation, vol. 58, no. 2-3, pp. 261-284, 2004.
[24] M. Papadopouli, H. Shen, E. Raftopoulos, M. Ploumidis, and F. Hernandez-Campos, "Short-term traffic forecasting in a campuswide wireless network," in Proc. of 16th Annual IEEE International Symposium on Personal Indoor and Mobile Radio Communications, Berlin, Germany, Sept. 2005.
[25] F. Chinchilla, M. Lindsey, and M. Papadopouli, "Analysis of wireless information locality and association patterns in a campus," in Proc. of IEEE Conference on Computer Communications (INFOCOM), Hong Kong, Mar. 2004.
[26] V. Paxson, "Empirically-derived analytic models of wide-area TCP connections," IEEE/ACM ToN, vol. 2, no. 4, pp. 316-336, Aug. 1994.
[27] A. B. Downey, "The structural cause of file size distributions," in Proc. of ACM SIGMETRICS, Cambridge, Massachusetts, USA, 2001, pp. 328329.

